Total number of printed pages – 7

63 (FY)SEM–2/MAJ2/PHYMAJ1024

2025

PHYSICS

(MAJOR)

Paper : PHYMAJ1024

(Mathematical Physics – I)

Full Marks : 50

Pass Marks : 20

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Choose the correct option from the following : ***(any five)*** 1×5=5

   (a) The complementary function of $(D^2 - 3D + 2)y = e^{4x}$ is

   (i) C.F = $A \sin x + B \cos 2x$

   (ii) C.F = $A \sin 2x + B \cos x$

   (iii) C.F = $Ae^{x} + Be^{2x}$

   (iv) C.F = $Ae^{2x} + Be^{x}$

(b) Consider the standard linear ordinary differential equation :

$$\frac{dy}{dx} + P(x)y = Q(x)$$

An integrating factor $\mu(x)$ is given by

(i) $\mu(x) = e^{\int Q(x)dx}$

(ii) $\mu(x) = e^{\int P(x)dx}$

(iii) $\mu(x) = \int P(x)dx$

(iv) $\mu(x) = \frac{1}{P(x)}$

(c) Given the differential equation

$$\frac{d^3y}{dx^3} + x\left(\frac{dy}{dx}\right)^2 - \sin y = 0,$$

what are the *order* and *degree* of this equation ?

(i) Order 3, Degree 1

(ii) Order 3, Degree 2

(iii) Order 2, Degree 1

(iv) Order 3, Degree not defined

(d) The Laplacian of a scalar $\Phi(r,\theta,\phi)$ in spherical coordinates is given by

$$\nabla^2\Phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Phi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Phi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\Phi}{\partial\phi^2}$$

Which of the following is the correct set of scale factors (h_r, h_θ, h_ϕ)?

(i) $(1, r, r\sin\theta)$

(ii) $(r, 1, \sin\theta)$

(iii) $(1, 1, 1)$

(iv) $(r^2, r, \sin\theta)$

(e) In a binomali distribution with parameters n (number of trials) and p (probability of success on a single trial), the variance is

(i) np

(ii) $np(1-p)$

(iii) $n(1-p)$

(iv) $p(1-p)$

2. Answer ***any five*** the following questions : 2×5=10

(a) Find the particular integral of $(D^2 - 3D + 2)y = \sin 3x$

(b) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$$

(c) A fair coin is tossed 5 times. What is the probability of getting exactly 3 heads ?

(d) Define orthogonal curvilinear coordinates. What are the scale factors for cylindrical coordinates (ρ, ϕ, z)?

(e) Determine the unit vector in the direction of a vector $\vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}$.

(f) If $\vec{a} = \hat{i} + \hat{j} - \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, $\vec{c} = \hat{i} - \hat{j} - \hat{k}$, find the vector $\vec{a} \times (\vec{b} \times \vec{c})$.

(g) Find the volume of the parallelepiped with adjacent sides $\overline{OA} = 3\hat{i} - \hat{j}$, $\overline{OB} = \hat{j} + 2\hat{k}$, $\overline{OC} = \hat{i} + 5\hat{j} + 4\hat{k}$.

3. Answer ***any five*** the following questions : 5×5=25

(a) Find the generfal solution for the differential equation :

$$\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$$

at $x = 0$.

(b) Check whether the functions $e^x \cos x$, $e^x \sin x$ are linearly independent or not.

(c) Find the divergence and curl of $\vec{V} = (xyz)\hat{i} + (3x^2y)j + (xz^2 - y^2z)\hat{k}$ at $(2, -1, 1)$.

(d) Prove that $(y^2 - z^2 + 3yz - 2x)\hat{i} + (3xz + 2xy)\hat{j} + (3xy - 2xz + 2z)\hat{k}$ is irrotational.

(e) Prove the relations given below :

(i) div curl $\vec{V} = 0$

(ii) $\text{div}(\phi\vec{V}) = \phi(\text{div}\,\vec{V}) + (\text{grad}\,\phi).\vec{V}$

(f) Prove that a cylindrical coordinate system is orthogonal.

(g) Find the square of the element of arc length (ds^2) and the volume element (dV) is spherical coordinates.

(h) (i) Write the probability density function (PDF) of a Gaussian distribution. Explain the roles of mean and the standard deviation.

(ii) The mean and the standard deviation of the heights of students in a class are 165 *cm* and 10 *cm* respectively. Find the probability that a randomly selected student has a height between 155 *cm* and 175 *cm*.

4. Answer ***any one*** of the following questions : 10×1=10

(a) (i) State Stokes theorem. Using Stokes theorem evaluate :

$$\oint_c (yz\,dx + zx\,dy + xy\,dz)$$

where c is the curve

$x^2 + y^2 = 1, z = y^2$ 1+4=5

(ii) State Gauss' divergence theorem. Using divergence theorem, find

$$\iint_s \vec{F}.\hat{n}\,ds$$

where $\vec{F} = 4xz\hat{i} - y^2 j + yz\hat{k}$ and S is the surface of the cube bounded by $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, $z = 1$. 1+4=5

(b) Find the solution of differential equation

$$x\frac{dy}{dx} + y = x^4$$

with the boundary condition that $y = 1$ at $x = 1$.
