63/1 (SEM–4) CC8/PHYHC4086

2025

PHYSICS

Paper : PHYHC4086

(Mathematical Physics–III)

Full Marks : 60

Pass Marks : 24

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Choose the correct answer from the following : ***(any five)*** 1×5=5

   (a) What is the value of $e^{3\pi i/2}$?

   (i) 1

   (ii) $-i$

   (iii) 1/2

   (iv) 0

(b) The Fourier Transform of $F\left[f(ax)\right]$ is

(i) $\frac{1}{a}F_s(s/a)$

(ii) $aF_s(s/a)$

(iii) $aF_s(a/s)$

(iv) $a/s\,F_s(s)$

(c) The Laplace Transform of $L(\sin hat)$ is

(i) a/s^2+a^2

(ii) a/s^2-a^2

(iii) s/s^2+a^2

(iv) s/s^2-a^2

(d) The Cauchy-Reimann Conditions are

(i) $\partial v/\partial x=\frac{\partial u}{\partial y}, \partial u/\partial x=\partial v/\partial y$

(ii) $\partial u/\partial x=-\frac{\partial v}{\partial y}, \partial v/\partial x=\partial u/\partial y$

(iii) $\partial u/\partial x=\partial v/\partial y, \frac{\partial v}{\partial x}=-\partial u/\partial y$

(iv) $\partial u/\partial x=-\partial v/\partial y, \partial v/\partial x=-\frac{\partial u}{\partial y}$

(e) The unit step function $u(t-a)$ is defined as

$$u(t-a)=0 \quad \text{when} \quad t<a$$
$$=1 \quad \text{when} \quad t\geq a$$

where $a\geq 0$.

The Laplace $L\left[u(t-a)\right]$ is

(i) e^{-as}

(ii) e^{-as}/s

(iii) e^{-as}/s^2

(iv) e^{-s}/s

(f) The complex conjugate of $2+3i/1-i$ is

(i) $1/2-5/2i$

(ii) $-1/2-5/2i$

(iii) $-1/2+5/2i$

(iv) $1/2+5/2i$

(g) The function of $1/(z-2)(z-3)$ has

(i) simple pole at $z=2$ and $z=3$

(ii) pole of order 2 at $z=2$

(iii) pole of order 2 at $z=3$

(iv) has no simple pole

(h) The Modulus of $-1-i$ is

(i) $\sqrt{2}$

(ii) $-\sqrt{2}$

(iii) 1

(iv) 2

(i) When the principal part of Laurent Series expansion is zero, then the singularity is called

(i) Essential singularity

(ii) Non-Essential singularity

(iii) Isolated singularity

(iv) Removable singularity

(j) The Fourier Transform of the Dirac-Delta Function $\delta(x-a)$ is

(i) $1/\sqrt{2\pi} \cdot e^{ika}$

(ii) e^{-ia}

(iii) 0

(iv) infinity

2. Answer the following questions : ***(any five)*** 2×5=10

(a) If $z_1 = 2-i$ and $z_2 = -2+i$, then find $Re(z_1 z_2 / \bar{z}_1)$.

(b) Find Laplace transformation of the functions
$f(t) = t^4 + t^2 + 1$.

(c) Find the pole and singularity of the function $e^z/(z-2)^3$.

(d) Find the inverse Laplace transform of $s/s+5$.

(e) What are the Fourier transforms of *sine* and *cosine*?

(f) Show that $F\{e^{iax} f(x)\} = F(s+a)$.

(g) Find the Laplace transform of $t \cos h\, at$.

3. Answer the following : ***(any five)*** 5×5=25

(a) Find the Taylor series expansion of the function $1/z+1$ about $z=1$.

(b) Find the Fourier transform of the function

$$f(t) = t \quad \text{for } |t| < a$$
$$= 0 \quad \text{for } |t| > a$$

(c) Use Cauchy's integral formula to calculate $\int_C \frac{2z+1}{z^2+1} dz$ where C is $|z| = 1/2$.

(d) Show that the Fourier transform of the convolution of $f(x)$ and $g(x)$ is the product of their Fourier transform

$$F[f(x) * g(x)] = F[f(x)] \cdot F[g(x)]$$

(e) Using Parseval's Identity prove that

$$\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt = \pi/2.$$

(f) Find the residue of the function at simple pole $\frac{z^2}{(z-1)^2 (z+2)}$.

(g) Find the square root of $(5 + 12i)$.

(h) Show that the following function is harmonic and determine its complex conjugate :

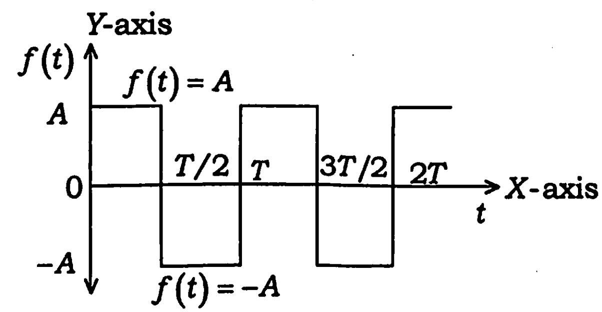
$$u = 2x(1-y)$$

(i) Evaluate the Integral

$$\int_0^{2\pi} \frac{1}{5-4\sin\theta} d\theta$$

4. Answer the following questions : ***(any two)*** 10×2=20

(a) Obtain the Laplace transformation of rectangular wave, given by



(b) Expand $f(z) = 1/(z+1)(z+2)$ in Laurent series for

(i) $|z| > 3$

(ii) $1 < |z| < 3$

5+5=10

(c) Solve the following equation by Laplace transform :

$$d^3y/dx^3 - 2\left(\frac{d^2y}{dx^2}\right) + 5(dy/dx) = 0$$

for $y = 0 \quad dy/dx = 1$ at $t = 0$

$y = 1$ at $t = \pi/8$

(d) *(i)* Write a short note on Modulus and Argument of a complex number.

2+3=5

(ii) Prove that : 2½×2=5

(a) $|z_1 - z_2| \geq |z_1| - |z_2|$

(b) $|z_1 + z_2| \leq |z_1| + |z_2|$