**63 (FY)SEM–3/MAJ/MATMAJ2024**

**2025**

# MATHEMATICS

Paper : MATMAJ2024

***[ Differential Equations ( ODE )]***

*Full Marks : 50*

*Pass Marks : 20*

Time : Two hours

***The figures in the margin indicate full marks for the questions.***

1. Choose the correct answer : 1×5=5

*(a)* The general solution of

$$\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 12y = 0$$ is

*(i)* $y = (c_1 + c_2x)e^{3x}$

*(ii)* $y = c_1e^{3x} + c_2e^{4x}$

(iii) $y = 0$

(iv) $y = (c_1 x + c_2 x^2) e^{4x}$

(b) The Wronskian of the functions $e^x$ and $e^{-x}$ are not zero is $W(e^x, e^{-x}) \neq 0$, then the functions $e^x$ and $e^{-x}$ are called

(i) linearly dependent

(ii) linearly independent

(iii) Non-linearly dependent

(iv) None of the above

(c) The degree of the differential equation

$$\sqrt[3]{1+\left(\frac{dy}{dx}\right)^4} = \frac{d^2y}{dx^2} \text{ is}$$

(i) 1

(ii) 2

(iii) 3

(iv) 4



(d) The solution of the differential equation

$$\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}} \text{ is}$$

(i) $\sin^{-1}x + \sin^{-1}y = c$

(ii) $\sin^{-1}x - \sin^{-1}y = c$

(iii) $\sin^{-1} y/x = c$

(iv) None of the above

(e) The differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0 \text{ is second order}$$

(i) linear homogeneous

(ii) non-linear

(iii) non-linear homogeneous

(iv) linear non-homogeneous

2. Answer the following questions : ***(any five)*** 2×5=10

(a) Find the Wronskian of the functions 1, $x$, $x^2$. Are they linearly independent?

(b) Solve : $(D^3 - 3D + 2)\, y = 0$

(c) Eliminate the constants from $ax^2 + by^2 = 1$.

(d) Examine whether the following differential equation is exact or not.

$$(2x^3 + 3y)dx + (3x + y - 1)dy = 0$$

(e) Find the integrating factor of the differential equation

$$\frac{dy}{dx} + xy = x^3.$$

(f) Draw an input-output compartmental diagram for radioactive nuclei.

(g) Formulate the mathematical model for density dependent growth.

3. Answer the following questions : ***(any five)*** 5×5=25

(a) Solve : $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 8e^{2x}$

(b) Solve : $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0$, when $x = 0$, $y = 0$ and $\frac{dy}{dx} = 0$

(c) Solve the Cauchy-Euler equation

$$x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 3y = x^2 logx.$$



(d) Solve : $p^2y + p(x - y) - x = 0, p = \frac{dy}{dx}$

(e) Solve : $(sinx\, cosy + e^{2x})dx + (cosx\, siny + tany)dy = 0$

(f) If the half-life of radioactive nuclei is $T$, then show that $K = \frac{log2}{T}$ for the problem of radioactive nuclei.

(g) Solve the logistic differential equation $\frac{dx}{dt} = \gamma x\left(1 - \frac{x}{k}\right)$ with the initial condition $x(0) = x$.

(h) Find the differential equation for epidemic model of influenza.

4. Answer the following question: ***(any one)*** 10×1=10

(a) (i) Apply the method of variation of parameter to solve 5

$$\frac{d^2y}{dx^2} + 4y = 4tan2x.$$

(ii) Solve the following differential equation using the method of undetermined coefficients 5

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = e^x \cos x.$$

(b) (i) Write the differential equation for the amount of salt in the tank at any time $t$ and solve it. 5

(ii) Solve : $x^2ydx - (x^3 + y^3)dy = 0$ 5