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63 (FY)SEM–2/MAJ2/MATMAJ1024

2025

MATHEMATICS

(MAJOR)

Paper : MATMAJ1024

(Calculus)

Full Marks : 70

Pass Marks : 28

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : 1×6=6

(i) Let $f(x) = |x|$ be a real valued function. Then

(A) f is not continuous at $x = 0$

(B) f is continuous at $x = 0$ but not differentiable at $x = 0$

(C) f is both differentiable and continuous at $x = 0$

(D) f is neither continuous nor differentiable at $x=0$.

(ii) Let $f(x)=x^2-1$ be defined on $[-1,1]$. Then which of the following is true for the curve $y=f(x)$?

(A) There exists a tangent at the point $(0,1)$ parallel to the x–axis

(B) There exists a tangent at the point $(0,1)$ parallel to the y–*axis*

(C) There exists a tangent at the point $(0,-1)$ parallel to the x–*axis*

(D) There exists a tangent at the point $(0,-1)$ parallel to the y–axis

(iii) Let f, g be two functions such that $\lim_{x\to a} f(x)=\lim_{x\to a} g(x)=0$, $f'(a), g'(a)$ exist and $g'(a)\neq 0$. Then

(A) $\lim_{x\to a}\frac{f(x)}{g(x)} \neq \lim_{x\to a}\frac{f'(x)}{g'(x)} =$

(B) $\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a} f'(x)$

(C) $\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a} g'(x)$

(D) $\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$

(iv) $\int_0^{\pi/2} \cos^6 x dx = ?$

(A) $\frac{5\pi}{8}$

(B) $\frac{5\pi}{16}$

(C) $\frac{5\pi}{32}$

(D) $\frac{5\pi}{64}$

(v) The angle of intersection of the curves $r=\sin\theta+\cos\theta$ and $r=2\sin\theta$ is

(A) $\pi/4$

(B) $\pi/2$

(C) $\pi/3$

(D) $\pi/6$

(vi) The radius of curvature of the parabola $y^2 = 4x$ at the vertex is

(A) $1/4$

(B) 4

(C) $1/2$

(D) 2

(vii) $\int_0^{\pi/2} \sin^6 \theta \cos^2 \theta d\theta = ?$

(A) $5\pi/64$

(B) $5\pi/256$

(C) $5\pi/128$

(D) $5\pi/32$

2. Answer the following questions : ***(any five)*** $2\times5=10$

(i) A function *f(x)* is defined as follows :

$$f(x) = \begin{cases} x \sin 1/x & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$$

Slow that f is continuous at $x = 0$.

(ii) Find $\frac{dy}{dx}$, if $y = \tan^{-1} \frac{\cos x}{1 + \sin x}$.

(iii) Give the geometrical interpretation of Lagrange's Mean Value Theorem.

(iv) Let $f(x)$ be defined on $[a,b]$ such that $f(x)$ is continuous in $[a,b]$ and $f'(x)$ exists in (a,b). If $f'(x) = 0$ for all $x \in (a,b)$, show that f is constant in $[a,b]$.

(v) If $r = a(1 + \cos\theta)$, then find $\frac{ds}{d\theta}$, where s denotes the arc length with usual meaning.

(vi) Find the centre of curvature of $xy = 12$ at the point (3,4).

(vii) Find the length of the arc of the curve $y = \log \sec x$ between $x = 0$ and $x = \pi/3$

3. Answer the following questions : ***(any six)*** 5×6=30

(i) Define derivability of a function $f(x)$ at a point $x=c$ in an interval. Show that the function

$$f(x)=\begin{cases}3+2x & \text{for } -3/2 < x \le 0\\ 3-2x & \text{for } 0 < x < 3/2\end{cases}$$

is continuous at $x=0$, but $f'(0)$ does not exist. 1+2+2=5

(ii) Find y_n, where $y=\dfrac{1}{x^2+a^2}$.

(iii) State and prove Cauchy's Mean Value Theorem.

(iv) Evaluate the following ***(any one)***:

(a) $\displaystyle\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{1/x^2}$

(b) $\displaystyle\lim_{x\to 0}\left(\frac{1}{x^2}-\frac{1}{\sin^2 x}\right)$

(v) Find the asymtotes of the following curve :

$$x^3-2y^3+xy(2x-y)+y(x-y)+1=0$$

(vi) If (α,β) be the coordinates of the centre of curvature of the parabola $\sqrt{x}+\sqrt{y}=\sqrt{a}$ at (x,y), then show that $\alpha+\beta=3(x+y)$.

(vii) Find the pedal equation of the curve $y^2=4ax$ with regard to its vertex.

(viii) If n is a positive integer, prove that

$$\int_0^{\pi/2}\sin^n dx=\begin{cases}\frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdots\frac{3}{4}\cdot\frac{1}{2}\cdot\frac{\pi}{2} & \text{if } n \text{ is even}\\ \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdots\frac{4}{5}\cdot\frac{2}{3}\cdot 1 & \text{if } n \text{ is odd}\end{cases}$$

(ix) The part of the parabola $y^2=4ax$ bonded by the latus rectum revolves about the tangent at the vertex. Find the area of the curved surface of the reel thus generated.

4. Answer the following question : ***(any two)*** 12×2=24

(i) *(a)* State and prove Leibnitz's Theorem to find the nth derivative of the product of two functions. 1+5=6

(b) Let $y=\sin(m\sin^{-1}x)$. Then show that $(1-x^2)y_2-2y_1+m^2y=0$

and $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2-n^2)y_n = 0$ 2+4=6

(ii) (a) Express $\log(1+x)\ \forall x \in [-1,1]$ as a power series of x by using Maclaurin's Theorem.

(b) Find $\frac{d}{dx}(\coth x)$.

(c) Verify Rolle's Theorem for the function $f(x) = x(x+3)e^{-x/2}$ in $[-3,0]$. 6+2+4=12

(iii) (a) If ρ_1 and ρ_2 be the radii of curvature at the ends P and D of conjugate diameters of the ellipse $x^2/a^2 + y^2/b^2 = 1$, then show that

$$\rho_1^{2/3} + \rho_2^{2/3} = (a^2+b^2)/(ab)^{2/3}$$

(b) Find the pedal equation of the curve $r^m = a^m \cos m\theta$.

(c) Prove that the curves

$r^n = a^n \cos n\theta$ and $r^n = b^n \sin n\theta$ cut orthogonally 6+3+3=12

(iv) (a) Show that for the ellipse

$x^2/a^2 + y^2/b^2 = 1$, the radius of curvature at an extremity of the major axis is equal to half of the latus rectum.

(b) Show that $\int_0^1 \frac{\log(1+x)}{1+x^2}dx = \frac{\pi}{8}\log 2$.

(c) Find the area above the x-axis included between the parabola $y^2 = ax$ and the circle $x^2+y^2 = 2ax$. 5+4+3=12