Total number of printed pages–11

63(FY) SEM-4/MIN/MATMIN2024

2025

MATHEMATICS

(Minor)

Paper : MATMIN2024

(Vector Calculus)

Full Marks : 70

Pass Marks : 28

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Choose the correct answer : $1 \times 10 = 10$

   (a) The value of a scalar triple product, if *two* of its vectors are equal is

   (i) One

   (ii) Two

   (iii) Zero

   (iv) Three

(b) The value of $\hat{i}.(\hat{j}\times\hat{k})+(\hat{i}\times\hat{k}).\hat{j}$ is

(i) 0

(ii) 1

(iii) $2\hat{k}$

(iv) $2\hat{j}$

(c) The volume of the parallelopiped whose coterminous edges are $2\hat{i}-3\hat{j}+\hat{k}$, $\hat{i}-\hat{j}+2\hat{k}$ and $2\hat{i}+\hat{j}-\hat{k}$ is

(i) 1

(ii) 0

(iii) 12

(iv) 14

(d) If $\overline{A}=t^2\hat{i}-t\hat{j}+(2t+1)\hat{k}$ and $\overline{B}=(2t-3)\hat{i}+\hat{j}-t\hat{k}$ then $\frac{d}{dt}(\overline{A}.\overline{B})$ at $t=1$ is

(i) 4

(ii) 6

(iii) – 4

(iv) – 6

(e) The gradient of a scalar function $\varphi(x,y,z)$ is

(i) $\frac{\partial^2\varphi}{\partial x^2}\hat{i}+\frac{\partial^2\varphi}{\partial y^2}\hat{j}+\frac{\partial^2\varphi}{\partial z^2}\hat{k}$

(ii) $\frac{\partial\varphi}{\partial x}+\frac{\partial\varphi}{\partial y}+\frac{\partial\varphi}{\partial z}$

(iii) $\frac{\partial\varphi}{\partial x}\hat{i}+\frac{\partial\varphi}{\partial y}\hat{j}+\frac{\partial\varphi}{\partial z}\hat{k}$

(iv) $\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}+\frac{\partial^2\varphi}{\partial z^2}$

(f) Which of the following is equivalent to $\nabla.(\vec{A} \times \vec{B})$?

(i) $\vec{B}.(\nabla \times \vec{A}) - \vec{A}.(\nabla \times \vec{B})$

(ii) $\vec{B} \times (\nabla.\vec{A}) - \vec{A} \times (\nabla.\vec{B})$

(iii) $\vec{B}.(\nabla.\vec{A}) - \vec{A}.(\nabla.\vec{B})$

(iv) $\vec{B}.(\nabla \times \vec{A}) - \vec{A}.(\nabla.\vec{B})$

(g) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then $\nabla.\vec{r} =$

(i) 0

(ii) 3

(iii) –3

(iv) 1

(h) If $\vec{f} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$, then the value of $\nabla.\vec{f}$ is

(i) $2(x+y+z)$

(ii) $x+y+z$

(iii) $x+2y+z$

(iv) $2x+y+2z$

(i) The curl of a vector function $\vec{F} = x^2\hat{i} + 2z\hat{j} - y\hat{k}$ is

(i) $-3\hat{k}$

(ii) $-3\hat{i}$

(iii) 0

(iv) $-3\hat{j}$

(j) By Green's throren $\oint_c (Mdx + Ndy) =$

(i) $\iint_R \left\{ \frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right\} dxdy$

(ii) $\iint_R \left\{ \frac{\partial N}{\partial y} - \frac{\partial M}{\partial x} \right\} dxdy$

(iii) $\iint_R \left\{ \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right\} dxdy$

(iv) $\iint_R \left\{ \frac{\partial M}{\partial x} - \frac{\partial N}{\partial y} \right\} dxdy$

2. Answer the following questions : ***(any five)*** 2×5=10

(a) Prove that

$$\left[\vec{a}\vec{b}\vec{c}\right] = \left[\vec{b}\vec{c}\vec{a}\right] = \left[\vec{c}\vec{a}\vec{b}\right]$$

(b) Prove that

$$\left[\vec{a}\left(\vec{b}+\vec{c}\right)\vec{d}\right] = \left[\vec{a}\vec{b}\vec{d}\right] + \left[\vec{a}\vec{c}\vec{d}\right]$$

(c) If $\vec{a}$ and $\vec{b}$ are differentiable vector functions of a scalar *t*, then show that

$$\frac{d}{dt}\left(\vec{a}+\vec{b}\right) = \frac{d\vec{a}}{dt} + \frac{d\vec{b}}{dt}$$

(d) A particle moves along the curve

$$x = 3t^2, \quad y = t^2 - 2t, \quad z = t^3$$

Find the velocity and acceleration at $t = 1$

(e) Prove that

$$\text{div}\left(curl\,\vec{F}\right) = 0, \text{ where } \vec{F} = x\hat{i} + y\hat{j} = z\hat{k}$$

(f) If $\varphi = x^3 + y^3 + z^3 - 3xyz$ find *div* (*grad* φ).

(g) Evaluate

$$\iint_S x^2 dydz + y^2 dzdx + 2z(xy - x - y)dxdy$$

where *S* is the surface of the cube

$$0 \le x \le 1, \quad 0 \le y \le 1, \quad 0 \le z \le 1$$

3. Answer the following questions : ***(any six)***

5×6=30

(a) Prove that the necessary and sufficient condition for a vector $\overline{f}(t)$ to have constant magnitude is that

$$\overline{f}.\frac{d\overline{f}}{dt}=0$$

(b) Find the constant P such that the vectors $\vec{a}=2\hat{i}-\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{c}=3\hat{i}+p\hat{j}+5\hat{k}$ are coplaner.

(c) Prove that

$$\vec{a}\times\left(\vec{b}\times\vec{c}\right)=\left(\vec{a}.\vec{c}\right)\vec{b}-\left(\vec{a}.\vec{b}\right)\vec{c}$$

(d) Established the identify.

$$\vec{a}=\left(\hat{i}.\vec{a}\right)\hat{i}+\left(\hat{j}.\vec{a}\right)\hat{j}+\left(\hat{k}.\vec{a}\right)\hat{k}$$

(e) If $\overline{a}=\left[\overline{p}\overline{q}\overline{r}\right]$, then show that

$$\frac{d\overline{a}}{dt}=\left[\frac{d\overline{p}}{dt}\overline{q}\overline{r}\right]+\left[\overline{p}\frac{d\overline{q}}{dt}\overline{r}\right]+\left[\overline{p}\overline{q}\frac{d\overline{r}}{dt}\right]$$

(f) Show that

$$curl\left(\overline{A}+\overline{B}\right)=curl\,\overline{A}+curl\,\overline{B}$$

(g) Find the unit normal to the surface

$2x^2y+3yz=4$ at the point $(1,-1,-2)$

(h) Evaluate $\int_c \overline{F}.d\overline{r}$ where $\overline{F}=x^2\hat{i}+y^3\hat{j}$ and C is the arc parabola $y=x^2$ is the xy – plane from $(0,0)$ to $(1,1)$

(i) If $\overline{F}$ is a conservative field, prove that $curl\ \overline{F}=\nabla\times\overline{F}=0$.

4. Answer the following questions : ***(any two)*** 10×2=20

(a) If $\vec{r} = \vec{a}\cos\mu t + b\sin\mu t$, where $\vec{a}$ and $\vec{b}$ are two constant vectors and μ is a constant scalar, prove that

$$\frac{d^2\vec{r}}{dt^2} + \mu\vec{r} = 0$$

Also, show that

$$\vec{r} \times \frac{d\vec{r}}{dt} = \mu\left(\vec{a} \times \vec{b}\right)$$

(b) If $\vec{a}$ is a constant vector, prove that

(i) $\nabla\left(\vec{a}.\vec{r}\right) = \vec{a}$

(ii) $\nabla.\left(\vec{a} \times \vec{r}\right) = 0$ 5+5=10

(c) State and prove the Green's theorem. 2+8=10

(d) Prove that

(i) $curl\left(\varphi\vec{A}\right) = \left(grad\,\varphi\right) \times \vec{A} + \varphi\, curl\,\vec{A}$

(ii) $div\left(grad\,\varphi\right) = \nabla^2\varphi$ 5+5=10