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63(FY) SEM-4/MAJ/MATMAJ2054

2025

**MATHEMATICS**

(Major)

Paper : MATMAJ2054

**(Analytical Geometry (2D) and Vector Calculus)**

Full Marks : 70

Pass Marks : 28

Time : Three hours

**The figures in the margin indicate  
full marks for the questions.**

1. Choose the correct answer :  $1 \times 6 = 6$

(a) If an equation  $hxy + gx + fy + c = 0$   
represents a pair of lines, then

(i)  $gh = cf$

(ii)  $fh = cg$

(iii)  $fg = ch$

(iv)  $hf = -cg$

(b) When the origin is shifted to  $(-1, 2)$  by the translation of axes, the transformed equation of  $x^2 + y^2 + 2x - 4y + 1 = 0$  is

(i)  $x'^2 + y'^2 = 4$

(ii)  $x'^2 + y'^2 = 16$

(iii)  $x'^2 + 2x' + y'^2 = 4$

(iv)  $x'^2 - 2x' + y'^2 = 16$

(c) Vector triple product  $\vec{a} \times (\vec{b} \times \vec{c})$  of the three vectors  $\vec{a}$ ,  $\vec{b}$  and  $\vec{c}$  is given by

(i)  $(\vec{a} \cdot \vec{b})\vec{c} - (\vec{a} \cdot \vec{c})\vec{b}$

(ii)  $(\vec{b} \cdot \vec{c})\vec{a} - (\vec{a} \cdot \vec{c})\vec{b}$

(iii)  $(\vec{b} \cdot \vec{c})\vec{a} - (\vec{a} \cdot \vec{b})\vec{c}$

(iv)  $(\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$

(d) If the pair of lines given by  $(x \cos \alpha + y \sin \alpha)^2 = (x^2 + y^2) \sin^2 \alpha$  are perpendicular to each other, then  $\alpha$  is

(i)  $0$

(ii)  $\frac{\pi}{2}$

(iii)  $\frac{\pi}{4}$

(iv)  $\frac{\pi}{6}$

(e) If vectors  $2\hat{i} - \hat{j} + \hat{k}$ ,  $\hat{i} + 2\hat{j} - 3\hat{k}$  and  $3\hat{i} + p\hat{j} + 5\hat{k}$  are coplanar, then the value of  $p$  is

(i)  $2$

(ii)  $-2$

(iii)  $-1$

(iv)  $-4$

(f) If  $\vec{r} = 3xz\hat{i} + 2xy\hat{j} - yz^2\hat{k}$ , then  $\text{div } \vec{r}$  is

(i)  $3x + 2y + 2z$

(ii)  $2x + 3y + 2z$

(iii)  $x + 2y + z$

(iv)  $3z + 2x - 2yz$

2. Answer the following questions : **(any five)**  
 $2 \times 5 = 10$

(a) What will be the form of  $x \cos \alpha + y \sin \alpha = P$ , when the axes are rotated through an angle  $\alpha$  ?

(b) Find the polar form of the equation  $(x^2 + y^2)^2 = a^2(x^2 - y^2)$

(c) Prove that  $\text{curl}(\text{grad } \phi) = 0$

(d) The co-ordinates of new origin are (2, 1) and the axes are rotated through an angle  $60^\circ$ . If the co-ordinates of a point in the new system are

$\left(\frac{3-4\sqrt{3}}{2}, -\frac{4+3\sqrt{3}}{2}\right)$ . Find the co-ordinate of it in the old system.

(e) Evaluate :  $(2\hat{i} - 3\hat{j}) \cdot [(\hat{i} + \hat{j} - \hat{k}) \times (3\hat{i} - \hat{k})]$

(f) Find the condition that the line  $lx + my = n$  is a tangent to the parabola  $y^2 = 4ax$ .

(g) If  $r$  be the distance of  $P(x, y, z)$  from the origin and  $\vec{r}$  be the position vector of  $P$  relative to the origin, show that  $\text{curl } \vec{r} = 0$ .

3. Answer the following questions : **(any six)**  
5×6=30

(a) Transform the equation

$$x^2 + 2xy \tan 2\alpha - y^2 = a^2 \sec 2\alpha$$

to the rectangular axes inclined at an angle  $\alpha$  to the old rectangular axes.

(b) If  $\vec{a}$ ,  $\vec{b}$  and  $\vec{c}$  are three non-coplanar vectors then, show that

$$[\vec{b} \times \vec{c} \quad \vec{c} \times \vec{a} \quad \vec{a} \times \vec{b}] = [\vec{a} \quad \vec{b} \quad \vec{c}]^2$$

(c) Prove that the locus of the point whose polar with respect to the ellipse  $x^2/a^2 + y^2/b^2 = 1$  subtends a right angle at the origin, is the ellipse

$$x^2/a^4 + y^2/b^4 = 1/a^2 + 1/b^2.$$

(d) Prove that

$$\text{curl}(\vec{u} \times \vec{v}) = \vec{u} \text{div } \vec{v} - \vec{v} \text{div } \vec{u} \\ + (\vec{v} \cdot \nabla) \vec{u} - (\vec{u} \cdot \nabla) \vec{v}$$

(e) Prove that the locus of poles of tangents to the hyperbola  $x^2 - y^2 = a^2$  with respect to the parabola  $y^2 = 4ax$  is the ellipse

$$x^2/a^2 + y^2/(4a^2) = 1.$$

(f) Find the equation of the normal at  $(a \cos \theta, b \sin \theta)$  to the ellipse

$$x^2/a^2 + y^2/b^2 = 1.$$

(g) Find  $\nabla \phi$  at  $(1, -2, -1)$  in the direction of  $2\hat{i} - \hat{j} - 2\hat{k}$ , where  $\phi = x^2yz + 4xz^2$ .

(h) Find the equation of the bisectors of the angles between the lines  $ax^2 + 2hxy + by^2 = 0$ .

(i) If  $\bar{a}$ ,  $\bar{b}$ ,  $\bar{c}$  are three non-coplanar vectors, then show that any vector  $\bar{r}$  can be expressed as

$$\bar{r} = \frac{[\bar{b} \bar{c} \bar{r}]\bar{a} + [\bar{c} \bar{a} \bar{r}]\bar{b} + [\bar{a} \bar{b} \bar{r}]\bar{c}}{[\bar{a} \bar{b} \bar{c}]}$$

4. Answer the following questions : **(any two)**  
12×2=24

(a) (i) Reduce the following equation to the standard form :

$$3x^2 - 6xy - 5y^2 - 6x + 22y - 17 = 0$$

(ii) The polar of a point with respect to the parabola  $y^2 = 4ax$  touches the parabola  $x^2 + 4by = 0$ ; Prove that the locus of the point is a rectangular hyperbola. 7+5=12

(b) (i) Show that the area of the triangle formed by the lines  $ax^2 + 2hxy + by^2 = 0$  and  $lx + my = 1$  is

$$\frac{\sqrt{h^2 - ab}}{am^2 - 2hlm + bl^2}$$

(ii) Find the equation of the pair of straight lines through the origin perpendicular to the pair  $ax^2 + 2hxy + by^2 = 0$ .

(iii) Choose the new origin  $(h, k)$  without changing the directions of the axes, such that the equation  $5x^2 - 2y^2 - 30x + 8y = 0$  may reduce to the form  $Ax'^2 + By'^2 = 1$ .

$$6+4+2=12$$

(c) (i) Define Green's theorem. Evaluate by Green's theorem. 6

$$\oint_C \{(\cos x \sin y - xy) dx + \sin x \cos y dy\}$$

where  $C$  is the circle  $x^2 + y^2 = 1$  in the  $xy$  - plane described in the positive sense.

(ii) Define Stokes' theorem. 6

$$\text{Evaluate : } \oint_C (e^x dx + 2y dy - dz)$$

by using Stokes' theorem, where  $c$  is the curve  $x^2 + y^2 = 4$ ,  $z = 2$ .

(d) (i) Find the equation of the tangent to the conic represented by the equation  $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$  at the point  $(x', y')$ .

- (ii) Prove that the locus of the poles of normal chords of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ is given by}$$

$$\frac{a^6}{x^2} + \frac{b^6}{y^2} = (a^2 - b^2)^2.$$

$$7+5=12$$

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