Total number of printed pages–8

63/1 (SEM–4) SEC2/MATSE4022

2025

MATHEMATICS

Paper : MATSE4022

(Vector Calculus)

Full Marks : 50

Pass Marks : 20

Time : Two hours

The figures in the margin indicate full marks for the questions.

1. Choose the correct option : ***(any five)*** $1\times5=5$

(a) $\vec{a}\times\left(\vec{b}+\vec{c}\right)+\vec{b}\times\left(\vec{c}+\vec{a}\right)+\vec{c}\times\left(\vec{a}+\vec{b}\right)$ is equal to

(i) $2\,[abc]$

(ii) 0

(iii) 3

(iv) None of these

(b) The area of the parallelogram whose adjacent sides are $2\hat{i}-3\hat{k}$ and $4\hat{j}+2\hat{k}$ is

(i) $2\sqrt{14}$

(ii) $4\sqrt{14}$

(iii) $16\sqrt{14}$

(iv) $\sqrt{14}$

(c) The value of p such that the vectors $\left(\hat{i}+3\hat{j}-2\hat{k}\right)$, $\left(2\hat{i}-\hat{j}+4\hat{k}\right)$ and $\left(3\hat{i}+2\hat{j}+p\hat{k}\right)$ are coplanar is

(i) 2

(ii) 4

(iii) 8

(iv) 10

(d) $\hat{i}\times\left(\vec{a}\times\hat{i}\right)+\hat{j}\times\left(\vec{a}\times\hat{j}\right)+\hat{k}\times\left(\vec{a}\times\hat{k}\right)=$

(i) a

(ii) $2a$

(iii) $3a$

(iv) 0

(e) The value of Curl $\vec{r}$ is

(i) r

(ii) $\hat{r}$

(iii) 0

(iv) None of the above

(f) The value of div$\left(\vec{r}\times\vec{a}\right)$ where $\vec{a}$ is a constant vector is

(i) a

(ii) $2a$

(iii) $-2a$

(iv) 0

(g) If $\vec{v}$ is a solenoidal vector, then div $\vec{v}$ =

(i) 0

(ii) 1

(iii) 2

(iv) 3

(h) The vector differential operator is

(i) $\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$

(ii) $\vec{\Delta} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$

(iii) $\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial x}$

(iv) $\vec{\nabla} = \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}$

(i) A vector whose divergence is zero is called

(i) Solenoidal vector

(ii) Irrational vector

(iii) Polar

(iv) Axial

(j) If Curl $\vec{a} = 0$, then $\vec{a}$ is

(i) Solenoidal

(ii) Irrational

(iii) Polar

(iv) Axial

2. Answer ***any five*** of the following questions : 2×5=10

(a) Prove that

$$\left[\vec{a}\times\vec{b},\ \vec{b}\times\vec{c},\ \vec{c}\times\vec{a}\right] = \left[\vec{a}\,\vec{b}\,\vec{c}\right]^2$$

(b) Show that the vectors

$\vec{a} - 2\vec{b} + 3\vec{c}$, $-2\vec{a} + 3\vec{b} - 4\vec{c}$ and

$\vec{a} - 3\vec{b} + 5\vec{c}$ are coplanar.

(c) If $\vec{f} = x^2 z\hat{i} - 2y^3 z^2 \hat{j} + xy^2 z\hat{k}$, find div $\vec{p}$ and curl $\vec{f}$ at (1, – 1, 1).

(d) A particle moves along the curve $x = t^3 + 1$, $y = t^2$, $z = 2t + 5$ where t is the time. Find the component of its velocity and acceleration at $t = 1$ in the direction $\hat{i} + \hat{j} + 3\hat{k}$.

(e) If $\vec{r} = \vec{a}.e^{nt} + \vec{b}e^{-nt}$ where $\vec{a}$ and $\vec{b}$ are the constant vector then prove that

$$\frac{d^2r^2}{dr} = n^2r^2$$

(f) If $\vec{f}$ and $\vec{g}$ are irrational, then show that $\vec{f} \times \vec{g}$ is a solenoidal vector.

(g) If $\vec{A} = \left(2x^2y - x^4\right)\hat{i} + \left(e^{xy} - y\sin x\right)\hat{j} + x^2\cos y\,\hat{k}$, verify $\frac{\partial^2 \vec{A}}{\partial y \partial x} = \frac{\partial^2 \vec{A}}{\partial x \partial y}$.

3. Answer ***any five*** of the following questions : 5×5=25

(a) Show that

$$\vec{a} \times \left(\vec{b} \times \vec{c}\right) + \vec{b} \times \left(\vec{c} \times \vec{a}\right) + \vec{c} \times \left(\vec{a} \times \vec{b}\right) = 0$$

(b) If $\vec{r} = (a\cos t)\,\hat{i} + (a\sin t)\,\hat{j} + (a\tan t)\,\hat{k}$ find $\left[\frac{d\vec{r}}{dt} \times \frac{d^2\vec{r}}{dt^2}\right]$.

(c) Find $\vec{r}$ from the equation, $\frac{d^2\vec{r}}{dt^2} = at + b$, given that both $\vec{r}$ and $\frac{d\vec{r}}{dt}$ vanish when $t = 0$.

(d) Evaluate : $\frac{d^2}{dt^2}\left[\vec{r}\,\frac{d\vec{r}}{dt} \cdot \frac{d^2\vec{r}}{dt^2}\right]$.

(e) Prove that :

$$div\left(\vec{f} \times \vec{g}\right) = \vec{g}\text{ . Curl }\vec{f} - \vec{f}\text{ .Curl }\vec{g}.$$

(f) Prove that :

$$div\left[\left(\vec{r} \times \vec{a}\right) \times \vec{b}\right] = -2\vec{b}.\vec{a}.$$

(g) If $\vec{f}$ = grad $\left(x^3 + y^3 + z^3 - 3xyz\right)$, then find $div\,\vec{f}$ and curl $\vec{f}$.

(h) Prove that :

$$\text{Curl}\left(\vec{u} \times \vec{v}\right) = \vec{u}.div\ \vec{v} - \vec{v}.div\ \vec{u}$$

$$+\left(\vec{v}.\nabla\right)\vec{u} - \left(\vec{u}.\nabla\right)\vec{v}.$$

(i) Prove that : $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$.

4. Answer ***any one*** of the following : 10×1=10

(a) If $\vec{r} = \vec{a}\cos\omega t + \vec{b}\sin\omega t$ show that

$$\vec{r} \times \frac{d\vec{r}}{dt} = \omega \vec{a} \times \vec{b} \text{ and } \frac{d^2\vec{r}}{dt^2} = -\omega^2 \vec{r}.$$

(b) Prove that :

$$\nabla^2(\phi\psi) = \phi\nabla^2\psi + 2\vec{\nabla}\phi\vec{\nabla}\psi + \psi\nabla^2\phi$$

(c) Find the value of constants *a*, *b* and *c* so that the directional derivatives of $\phi = axy^2 + byz + cz^2x^3$ at (1, 2, – 1) has a maximum magnitude 64 in a direction parallel to *z*-axis.
