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MATHG 4046/MATRC 4046

2025

**MATHEMATICS**

Paper : MATHG4046/MATRC4046

**( Differential Equations )**

Full Marks : 80

Pass Marks : 32

Time : Three hours

**The figures in the margin indicate  
full marks for the questions.**

1. Choose the correct options from the following : **(any six)** 1×6=6

তলত দিয়াবোৰৰ সঠিক বিকল্পটো বাছি উলিওৱা :

(যিকোনো ছয়টা)

- (a) The differential equation  $Mdx + Ndy = 0$  will be exact, if

$Mdx + Ndy = 0$  অৱকল সমীকৰণটো যথার্থ হ'ব, যদি

(i) 
$$\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$$

$$(ii) \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$(iii) \frac{\partial N}{\partial x} \neq \frac{\partial M}{\partial y}$$

$$(iv) \frac{\partial N}{\partial y} \neq \frac{\partial M}{\partial x}$$

(b) The equation  $\frac{dy}{dx} + Py = Q$  (where  $P$  and  $Q$  are functions of  $x$  alone) is a

$$\frac{dy}{dx} + Py = Q \text{ (যদি } P \text{ আৰু } Q \text{ কেৱল } x\text{-ৰ ফলন)}$$

সমীকৰণটো এটা

(i) Linear differential equation

ৰৈখিক অৱকল সমীকৰণ

(ii) Bernoulli's linear differential equation

বাৰ্নলীৰ ৰৈখিক অৱকল সমীকৰণ

(iii) Exact differential equation

যথার্থ অৱকল সমীকৰণ

(iv) Clairauts equation

ক্লেইবট সমীকৰণ

(c) The integrating factor of the following differential equation is

তলৰ অৱকল সমীকৰণটোৰ অনুকলন উৎপাদকটো হ'ল

$$\frac{dy}{dx} + Py = Q$$

(where  $P, Q$  are functions of  $x$  alone or constants)

(য'ত  $P, Q$  হ'ল কেৱল  $x$ -ৰ ফলন বা ধ্ৰুৱক)

$$(i) e^{\int P dx}$$

$$(ii) e^{\int Q dx}$$

$$(iii) e^{\int P dy}$$

$$(iv) e^{\int Q dy}$$

(d) The Wronskian of the set  $\{x^2, x^2 \log x\}$  is

$\{x^2, x^2 \log x\}$  ফলনটোৰ Wronskian হ'ল

$$(i) x$$

$$(ii) x^2$$

$$(iii) x^3$$

$$(iv) x^4$$

(e) The order and degree of the following differential equation are

তলৰ অৱকল সমীকৰণটোৰ ক্ৰম আৰু ঘাত হ'ল

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x$$

(i) 1, 1

(ii) 1, 2

(iii) 2, 2

(iv) 2, 1

(f)  $\frac{y dx - x dy}{y^2} = ?$

(i)  $d\left(\frac{y}{x}\right)$

(ii)  $d\left(\frac{y^2}{x}\right)$

(iii)  $d\left(\frac{x}{y}\right)$

(iv)  $d\left(\frac{x}{y^2}\right)$

(g) The Lagrange's linear equation is

লাগ্ৰাঞ্জৰ ৰৈখিক সমীকৰণটো হ'ল

(i)  $P_p + Q_q = R$

(ii)  $P_p + Q_q = PQ$

(iii)  $P_q + Q_p = R$

(iv)  $P_p \cdot Q_q = R$

where  $P, Q, R$  are the functions of  $x, y, z$  respectively.

য'ত  $P, Q, R$  ক্ৰমে  $x, y, z$ -ৰ ফলন।

(h) Which of the following is a linear partial differential equation ?

তলৰ কোনটো এটা ৰৈখিক আংশিক অৱকল সমীকৰণ ?

(i)  $p^2 + q^2 = 1$

(ii)  $p + q = pq$

(iii)  $x^2 p^2 + y^2 q^2 = z^2$

(iv)  $x^2 p + y^2 q = z^2$

- (i) The following partial differential equation will be parabolic, if

তলৰ আংশিক অৱকল সমীকৰণটো অধিবৃত্তীয় হ'ব, যদি

$$A \frac{\partial^2 u}{\partial x^2} + B \frac{\partial^2 u}{\partial x \partial y} + C \frac{\partial^2 u}{\partial y^2} = 0$$

- (i)  $B^2 - 4AC > 0$   
(ii)  $B^2 - 4AC < 0$   
(iii)  $B^2 - 4AC = 0$   
(iv)  $B^2 - 4AC \neq 0$
- (j) The form of the Clairaut's equation is

ক্লেইৰাউট সমীকৰণটোৰ ৰূপটো হ'ল

- (i)  $\frac{dy}{dx} + Py = Q$   
(ii)  $Mdx + Ndy = 0$   
(iii)  $f(x, y, z, p, q) = 0$   
(iv)  $y = Px + f(P)$

2. Answer **any five** from the following :

2×5=10

তলৰ যিকোনো পাঁচটাৰ উত্তৰ দিয়া :

- (a) Show that the following equation is exact

দেখুৱা যে তলৰ সমীকৰণটো যথার্থ

$$(2x^3 + 4y) dx + (4x + y - 1) dy = 0$$

- (b) Solve (সমাধান কৰা) :

$$(D^2 + 4) y = 0$$

- (c) Test the linear independence of the functions  $e^x \sin x$  and  $e^x \cos x$ .

$e^x \sin x$  আৰু  $e^x \cos x$  ফলন দুটাৰ বৈখিক স্বতন্ত্রতাৰ পৰীক্ষা কৰা।

- (d) Solve (সমাধান কৰা) :

$$e^x \tan y dx + (1 - e^x) \sec^2 y dy = 0$$

- (e) Form the partial differential equation by eliminating  $a, b$  from

$$Z = ax + by + a^2 + b^2$$

(where  $a$  and  $b$  are arbitrary constants)

$$Z = ax + by + a^2 + b^2, \text{ (য'ত } a \text{ আৰু } b \text{ স্বচ্ছ$$

ধ্ৰুৱক)-ৰ পৰা }  $a, b$  অপনয়ন কৰি আংশিক অৱকল সমীকৰণটো গঠন কৰা।

(f) Define order and degree of a partial differential equation.

আংশিক অৱকল সমীকৰণৰ ক্ৰম আৰু ঘাতৰ সংজ্ঞা লিখা।

(g) Convert the Bernoulli's equation

$x \frac{dy}{dx} + y = y^2 \log x$  to the linear differential equation.

$x \frac{dy}{dx} + y = y^2 \log x$  বাৰ্নলীৰ সমীকৰণটোৰ ৰৈখিক অৱকল সমীকৰণলৈ ৰূপান্তৰ কৰা।

3. Solve **any six** from the following :

$$5 \times 6 = 30$$

তলৰ যিকোনো ছয়টাৰ সমাধান কৰা :

(a)  $(3x^2 + 4xy) dx + (2x^2 + 2y) dy = 0$

(b)  $\frac{dy}{dx} + y \sec x = \tan x$

(c)  $\cos^2 x \frac{dy}{dx} + y = \tan x$

(d)  $\frac{dy}{dx} + y = xy^3$

(e)  $\frac{d^2y}{dx^2} - 3 \frac{dy}{dx} + 2y = e^x$

(f)  $\frac{d^2y}{dx^2} - 4y = e^x \sin 2x$

(g)  $x^2 p + y^2 q = z^2$

(h)  $y = 2px - p^2$

(i)  $pq = xy$

(j)  $p \tan x + q \tan y = \tan z$

4. Solve **any two** from the following :

$$10 \times 2 = 20$$

তলৰ যিকোনো দুটাৰ সমাধান কৰা :

(a)  $x^2 \frac{d^2y}{dx^2} - 4x \frac{dy}{dx} + 6y = x$

(b)  $\frac{dx}{dt} + 2x - 3y = t; \frac{dy}{dx} - 3x + 2y = e^{2t}$

(c)  $2xz - px^2 - 2qxy + pq = 0$

(d)  $z = px + qy + p^2 + q^2$

5. Answer **any one** from the following : 14

তলৰ যিকোনো এটাৰ উত্তৰ দিয়া :

(a) (i) Solve by the method of variation of parameters : 8

Variation of parameter পদ্ধতি ব্যৱহাৰ কৰি সমাধান কৰা :

$$\frac{d^2y}{dx^2} + y = \tan x$$

(ii) Solve (সমাধান কৰা) : 6

$$\frac{dx}{y-z} = \frac{dy}{z-x} = \frac{dz}{x-y}$$

(b) (i) Find the integral surface of the partial differential equation  $x^2p + y^2q + z^2 = 0$  which passes through the hyperbola  $xy = x + y, z = 1$ . 8

$x^2p + y^2q + z^2 = 0$  আংশিক অৱকল সমীকৰণটোৰ অনুকলনীয় পৃষ্ঠ নিৰ্ণয় কৰা যিয়ে  $xy = x + y, z = 1$  পৰাবৃত্তটোৰ মাজেৰে যায়।

(ii) Show that  $e^{\int P dx}$  is an integrating factor of the linear differential equation  $\frac{dy}{dx} + Py = Q$ ; (where  $P, Q$  are functions of  $x$ -alone). 6

দেখুৱা যে  $\frac{dy}{dx} + Py = Q$ ; (য'ত  $P, Q$  হ'ল কেৱল  $x$ -ৰ ফলন) বৈখিক অৱকল সমীকৰণটোৰ অনুকলন উৎপাদকটো  $e^{\int P dx}$ ।

(c) (i) Using Wronskian show that  $x, x^2, x^4$  are linearly independent. Determine the differential equation with these independent solutions. 3+5=8

Wronskian-ৰ সহায়ত দেখুৱা যে  $x, x^2, x^4$  বৈখিকভাৱে স্বতন্ত্ৰ। এই স্বতন্ত্ৰ সমাধানৰ সহায়ত অৱকল সমীকৰণটো নিৰ্ণয় কৰা।

(ii) Solve (সমাধান কৰা) : 6

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = \sin 2x$$