63 (FY)SEM–1/MAJ/MATMAJ1014

2025

MATHEMATICS

(Major)

Paper : MATMAJ1014

(Foundation of Mathematics)

Full Marks : 50

Pass Marks : 20

Time : Two hours

The figures in the margin indicate full marks for the questions.

1. Choose the correct answer : 1×5=5

   (a) If a is a non–zero integer then the gcd of a and 0 is

   (i) a

   (ii) $|a|$

   (iii) 0

   (iv) $-a$

(b) If $B=\{1,2,3\}$ then the relation $S=\{(1,1),(3,3),(2,2)\}$ on B is

(i) reflexive and symmetric but not transitive

(ii) reflexive and transitive but not symmetric

(iii) reflexive but neither transitive nor symmetric

(iv) reflexive, symmetric and transitive

(c) If an algebraic equation with real coefficient has the root $2+3i$ then it has also the root

(i) $3-2i$

(ii) $-2-3i$

(iii) $2-3i$

(iv) $-2+3i$

(d) If A and B are two $n\times n$ invertible matrices then

(i) $(AB)^{-1}=A^{-1}B^{-1}$

(ii) $(AB)^{-1}=AB$

(iii) $(AB)^{-1}=B^{-1}A^{-1}$

(iv) $(AB)^{-1}\neq B^{-1}A^{-1}$

(e) The nth roots of unity are

(i) $\cos\frac{2k\pi}{n}+i\sin\frac{2k\pi}{n}(k=0,1,2,\dots,n-1)$

(ii) $\cos\frac{4k\pi}{n}+i\sin\frac{4k\pi}{n}(k=0,1,2,\dots,n-1)$

(iii) $\cos\frac{k\pi}{n}+i\sin\frac{k\pi}{n}(k=0,1,2,\dots,n-1)$

(iv) $\cos\frac{3k\pi}{n}+i\sin\frac{3k\pi}{n}(k=0,1,2,\dots,n-1)$

2. Answer the following questions : ***(any five)*** 2×5=10

(a) The function $f:\mathbb{R}\to\mathbb{R}$ is defined as $f(x)=x^2+x+1,\ x\in\mathbb{R}$. Examine if f is onto.

(b) If α,β,γ are the roots of the equation $x^3+px^2+qx+r=0$, find the value of $\alpha^2+\beta^2+\gamma^2$.

(c) Let a,b,x,y be integers such that $a\equiv x\ (mod\ n)$ and $b\equiv y\ (mod\ n)$, where n is a natural number. Show that $ab\equiv xy\,(mod\ n)$.

(d) Determine if the following system of linear equations is consistent.

$$x-2y+z=0$$
$$2y-8z=8$$
$$5x-5z=10$$

(e) Let A be a 3×3 matrix. If u and v are vectors in $\mathbb{R}^3$, prove that $A(u+v)=Au+Av$.

(f) Find all values of $(-i)^{\frac{1}{3}}$.

(g) Convert 21469 to octal and hexadecimal notation.

3. Answer the following questions : ***(any five)*** 5×5=25

(a) Find all the integral roots of $x^4+4x^3+8x+32=0$ by Newton's method.

(b) If the functions $f:x\to y$ and $g:Y\to Z$ are both surjective then show that go f is also surjective.

(c) Let a and b be two natural numbers. Prove that there are unique non–negative integers q and r, with $0\le r<b$, such that $a=qb+r$.

(d) Row reduce the following matrix to reduced echelon from. Also circle the pivot positions in the final matrix.

$$\begin{bmatrix}1&2&3&4\\4&5&6&7\\6&7&8&9\end{bmatrix}$$

(e) Solve $x^4 - 4x^3 + 4x - 1 = 0$, one root being $2 + \sqrt{3}$.

(f) Prove that the equivalence classes of an equivalence relation on a set X induces a partition of X.

(g) Balance the following unbalanced chemical equation using vector equation approach.

$$Na_3PO_4 + Ba(NO_3)_2 \rightarrow Ba_3(PO_4)_2 + NaNO_3$$

(h) For any two non–zero integers a and b, prove that $\gcd(a,b) \text{lcm}(a,b) = |ab|$.

4. Answer the following questions : ***(any one)*** $10 \times 1 = 10$

(a) (i) If $a = \cos\frac{2r\pi}{n} + i\sin\frac{2r\pi}{n}$ and if r and p are prime to n, prove that $1 + a^p + a^{2p} + \ldots + a^{(n-1)p} = 0$. 5

(ii) Let $f : X \rightarrow Y$ be a map and P, Q be two subsets of Y. Prove that $f^{-1}(P \cup Q) = f^{-1}(P) \cup f^{-1}(Q)$. 5

(b) (i) Let A be a square $n \times n$ matrix. Prove that A is an invertible matrix if and only if A is row equivalent to the $n \times n$ identity matrix. 7

(ii) Suppose a and b are relatively prime integers and c is an integer such that $a | c$ and $b | c$. Prove that $ab | c$. 3

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