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63/1(SEM-5) DSE1/PHYHE5016

2024

PHYSICS

Paper : PHYHE5016

(Advanced Mathematical Physics-I)

Full Marks : 60

Pass Marks : 24

Time : 3 hours

The figures in the margin indicate full marks for the questions.

1. Choose the correct option from the following: ***(any five)*** 1×5=5

   a. Which one of the following is not a group? 1

   I. $X*Y=X+Y\sqrt{3}$

   II. $X*Y=X+Y$

   III. $A*B=A+B-AB$

   IV. $A*B=A+B+1$

b. Let $V_1=(1,-1,0)$, $V_2=(0,1,-1)$, $V_3=(0,0,1)$ be elements of R3. The set of vectors $\{V_1,V_2,V_3\}$ is 1

I. Linearly independent

II. Linearly dependent

III. Null

IV. None of these.

c. IF $X=(0,-5,6)$ and $Y=(4,7,3)$, then $d(X,Y)=$ 1

I. -13 II. 169

III. 13 IV. -17

d. The trace of the matrix $A=\begin{bmatrix}2 & 4\\ 3 & 7\end{bmatrix}$ is _

I. 2 II. 7

III. 9 IV. 5

e. The Eigen values of the matrix $\begin{bmatrix}3 & 0\\ 0 & 2\end{bmatrix}$ is

I. 3 & 0 II. 3 & 2

III. 0 & 2 IV. 0 & 0

f. If a square matrix U follow the relation $(\bar{U})^T=U^{-1}$ then U is

I. Orthogonal II. Unitary

III. Symmetric IV. Hermitian

g. The number of components for a tensor of rank 3 in three dimensional space is

I. 3 II. 9

III. 27 IV. 81

h. The rank and type of moment of inertia tensor is 1

I. 1 and symmetric

II. 2 and symmetric

III. 1 and anti-symmetric

IV. 2 and anti-symmetric

i. Which one of the following is the correct representation of dot product of two vectors $\vec{A}$ and $\vec{B}$ in tonsorial notation? 1

I. A^iB_j II. A^iB_i

III. A^iB^j IV. A_iB_j

Contd.

j. The rank of the mixed tensor A_k^{ijk} is

I. 1 II. 2

III. 3 IV. 4

2. Answer *any five* of the following questions : 2×5=10

a. Check whether the following vector form a basis of R^3 or not (1,1,2), (1,2,5), (5,3,4).

b. Evaluate the Eigen values for the given matrix $A=\begin{bmatrix} 2+3i & 4-5i \\ 3 & 4i \end{bmatrix}$

c. Show that $A=\begin{bmatrix} i & 0 \\ 0 & 1 \end{bmatrix}$ is a unitary matrix.

d. If A is a Hermitian matrix, show that e^{iA} is unitary.

e. Write the transformation relation for th tensor A^i_{jk}.

f. Define isotropic tensor. Give an example of an isotropic tensor.

g. Consider a tensor A_{st}^{pqr} is symmetric w.r.t indices p and q in one coordinate system. Show that it remain symmetric w.r.t p and q in any coordinate system.

3. Answer *any five* of the following questions : 5×5=25

a. State the condition for linear dependence of vector in R^N space. Are the vectors X_1 = 1,0,0), X_2 = (0,1,0) and X_3 = (0,0,1) 2+3

b. The standard basis for a 2×2 matrix

$$B=\left\{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix},\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix},\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right\},$$

and the basis

$$A=\left\{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},\begin{bmatrix} 2 & 0 \\ -1 & 0 \end{bmatrix},\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix},\begin{bmatrix} -3 & 0 \\ 0 & 2 \end{bmatrix}\right\},$$

(i) Find the transition matrix from *A* to *B*.

(ii) Find the matrix that has coordinate vector $[V]_A=(-8,3,5,-2)$

c. Show that every *n* dimensional vector space over the field *F* is isomorphic to the space F^n.

d. Using Cayley - Hamilton theorem, evaluate the inverse of the matrix

$$\begin{bmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$$

Contd.

e. Deduce the transformation relation for component of a contravariant tensor A^p in three dimensional space. 5

f. If $\alpha_1, \alpha_2, \alpha_3$ and $\beta_1, \beta_2, \beta_3$ are the corresponding tangent vectors and normal vectors in curvilinear coordinate system (u_1, u_2, u_3) then show that $\vec{\alpha}_i \vec{\beta}_j = \delta_{ij}$.

g. Using the vector identity $(\vec{a} \times \vec{b}).(\vec{c} \times \vec{b}) = (\vec{a} \times \vec{c}).(\vec{b} \times \vec{d}) - (\vec{a} \times \vec{d}).(\vec{b} \times \vec{c})$, deduce the relation between alternate tensor and Kronecker delta tensor. Hence show that $\in_{pqr} \in^{pqr} = 6$.

h. Suppose $ds^2 = g_{ij} dx^i dx^j$ is an invariant. Show that g_{ij} is a symmetric covariant tensor of rank two. Hence show that $g^{ij} g_{jl} = \delta^i_l$ 3+2

i. Determine the metric tensor g_{ij} in cylindrical coordinate system and hence find its conjugate tensor: 2+3=5

4. Answer *any two* of the following question: 10×2=20

(a) Find the Eigen values and the corresponding eigen vectors for the matrix 2+3=5

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}.$$ Hence find the matrix P such that $P^{-1}AP$ is a diagonal matrix

(b) Solve the following general system x^i of differential equation using matrix method

$$\frac{dx_1}{dt} = x_1 + x_2 \;\&\; \frac{dx_2}{dt} = 4x_1 + x_2$$ 10

(c) State Quotient law of tensor. 2+8

(d) Suppose in the coordinate system, a quantity $A(p,q,r)$ is defined by $A(p,q,r) B^{qs}_r = C^s_p$, where B^{qs}_r is an arbitary tensor and C^s_p is a tensor. Prove that $A(p,q,r)$ is also a tensor.

d. Using tensor formulism, evaluate $\vec{\nabla}.\vec{A}$ and $\nabla^2\varphi$ in cylindrical coordinate system (p,ϕ,Z). 6+4
