Total number of printed pages–8

63/1 (SEM–1) CC1/PHYHC1016

2024

PHYSICS

Paper : PHYHC1016

(Mathematical Physics–I)

Full Marks : 60

Pass Marks : 24

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Choose the correct option : ***(any five)*** : 1×5=5

   (a) Find the constant a so that the vector $\vec{A} = (x+3y)\,\hat{i} + (2y+3z)\,\hat{j} + (x+az)\,\hat{k}$ is solenoidal :

   (i) $a=-3$

   (ii) $a=-2$

   (iii) $a=4$

   (iv) $a=1$

Contd.

(b) The value for $\vec{\nabla}\times(\vec{\nabla}\phi)$ is—

(i) 0

(ii) 1

(iii) >0

(iv) <0

(c) Which is the correct statement?

(i) $\int_{-\infty}^{\infty}\delta(x-a)\,dx=1$

(ii) $\int_{-\infty}^{\infty}\delta(x+a)\,dx=1$

(iii) $\int_{\infty}^{-\infty}\delta(x-a)\,dx=1$

(iv) $\int_{\infty}^{-\infty}\delta(x+a)\,dx=1$

(d) Find y_2 if $y=e^{3x+2}$:

(i) $3e^{3x+2}$

(ii) $6y$

(iii) $4y$

(iv) $9y$

(e) If a is a simple constant, which is the derivative of $y=x^a$ from the following:

(i) ax^{a-1}

(ii) $(a-1)^x$

(iii) x^{a-1}

(iv) ax

(f) The given equation $(y^2-2x)dx+2xydy=0$ is exact when—

(i) $\frac{\partial P}{\partial y}=\frac{\partial Q}{\partial x}$

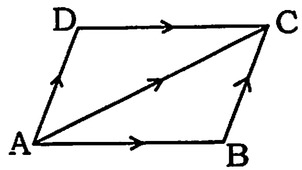
(ii) $\frac{\partial P}{\partial y}\neq\frac{\partial Q}{\partial x}$

(iii) $\frac{\partial P}{\partial y}=0$

(iv) $\frac{\partial Q}{\partial x}=0$

Here $P=y^2-2x$, $Q=2xy$

(g) Sum of two vectors $\overrightarrow{AB}$ and $\overrightarrow{BC}$ as shown in figure is given by—



(i) $\overrightarrow{BC}$

(ii) $\overrightarrow{CD}$

(iii) $\overrightarrow{AC}$

(iv) $\overrightarrow{AB}$

(h) Laplacian ∇^2 for a scalar function ϕ is

(i) $\nabla^2\phi$

(ii) $\vec{\nabla}\phi$

(iii) $\phi\vec{\nabla}$

(iv) $\phi\nabla^2$

(i) If $f(x, y) = 2x^3 - x^2y^6$, then the value of $\frac{\partial f}{\partial y}$ is—

(i) $6x^2 - 2xy^6$

(ii) $6x^2 - 2x$

(iii) $2x^3 - 12xy^5$

(iv) $-6x^2y^5$

(j) The order and degree of the differential equation $\frac{d^2y}{dx^2} + x^2\left(\frac{dy}{dx}\right)^2 = 0$ is —

(i) 2 and 1

(ii) 1 and 2

(iii) 2 and 2

(iv) 2 and 3

2. Answer ***any five*** of the following questions : 2×5=10

(a) Show that $\vec{F} = (xy + z^3)\hat{i} + x^2\hat{j} + 3xz^2\hat{k}$ is an irrotational field.

(b) If $\phi = 3x^2y - y^4z^2$ is a scalar field, find $\vec{\nabla}\phi$ at the point $(1, 2, 1)$. 2

Contd.

(c) Express the gradient operator $\vec{\nabla}$ in spherical polar co-ordinates.

(d) State the Gauss's divergence theorem.

(e) Show that

$$\delta(ax) = \frac{1}{|a|}\delta(x)$$

for any non-zero value of a.

(f) Express $\vec{\nabla} \times \vec{A}$ in spherical co-ordinates.

(g) If $\vec{V} = (3x^2 + 6y)\,\hat{i} - 14yz\hat{j} + 20xz^2\hat{k}$, then evaluate $\int \vec{V} \cdot \vec{dr}$, where C is a straight line joining (0, 0, 0) and (1, 1, 1).

3. Answer ***any five*** of the following questions : 5×5=25

(a) Find a unit outward normal drawn to the surface of the paraboloid of revolution $z = x^2 + y^2$ at the point $(1, 2, 5)$.

(b) Express $\nabla^2\psi$ in orthogonal curvilinear co-ordinates.

(c) How will you define divergence and curl of a vector $\vec{V}$? Evaluate $\vec{\nabla} \cdot \vec{r}$ and $\nabla \times \vec{r}$. 2+3=5

(d) Find the volume of a tetrahedron with $\vec{a}, \vec{b}, \vec{c}$ as adjacent edges (w.r.t. right-handed cartesian co-ordinates), where

$$\vec{a} = \hat{i} + 2\hat{k} \qquad b = 4\hat{i} + 6\hat{j} + 2\hat{k}$$

$$\vec{c} = 3\hat{i} + 3\hat{j} - 6\hat{k}$$

(e) Calculate the scale factors of cylindrical co-ordinate system.

(f) Define mean and standard deviation. 2½+2½=5

(g) Prove the following for Dirac-delta function :

$$\int_{-\infty}^{\infty} \delta(x-a)\, f(x)\,dx = f(a)$$

(h) Find the scalar potential of

$$\vec{A} = (y^2 \cos x + z^3)\,\hat{i} + (2y \sin x - 4)\,\hat{j} + (3xz^2 + 2)\hat{k}$$

(i) Write a short note on Gaussian distribution.

4. Answer ***any two*** of the following questions : 10×2=20

(a) Verify Stokes' theorem for $\vec{F} = (2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$, where S is the upper half surface of sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary.

(b) Obtain an expression for the curls of $\vec{A}$ is cylindrical co-ordinates.

(c) (i) Find constants *a, b* and *c* so that

$$V = (-4x - 3y + az)\hat{i} + (bx + 3y + 5z)\hat{j} + (4x + cy + 3z)\hat{k}$$ is irrotational.

(ii) Show that V can be expressed as the gradient of a scalar function. 5+5=10

(d) Solve the following differential equations : 5+5=10

(i) $\dfrac{dy}{dx} + xy = 2x$

(ii) $x\dfrac{dy}{dx} + y = x^3 + x$
