63(FY)SEM-2/MAJ2/PHYMAJ1024

2024

PHYSICS

Paper : PHYMAJ1024

(Mathematical Physics–I)

Full Marks : 50

Pass Marks : 20

Time : Two hours

The figures in the margin indicate full marks for the questions.

1. Choose the correct option from the following: $1\times5=5$

   (a) Which of the following is not a hypothesis testing method

   (i) Chi-square test

   (ii) Correlation test

   (iii) t-test

   (iv) z-test

Contd.

(b) The Taylor's series expansion of sin x is

(i) $1-\frac{x^2}{2!}+\frac{x^4}{4!}\ldots\ldots$

(ii) $1+\frac{x^2}{2!}+\frac{x^4}{4!}\ldots\ldots$

(iii) $1-\frac{x^3}{3!}+\frac{x^5}{5!}\ldots\ldots$

(iv) $1+\frac{x^3}{3!}+\frac{x^5}{5!}\ldots\ldots$

(c) The order and degree of the differential equation $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2+3y=0$ is

(i) Order 2, degree 2

(ii) Order 2, degree 1

(iii) Order 1, degree 2

(iv) Order 1, degree 1

(d) The value of λ for which the vector $\vec{A}=(x+3y)\hat{i}+(y-3z)\hat{j}+(x+\lambda z)\hat{k}$ is solenoidal is

(i) −2

(ii) 1

(iii) 2

(iv) 3

(e) The equation $e^x\frac{dy}{dx}+3y=x^2y$ is

(i) Separable and non-linear

(ii) Linear and not separable

(iii) Both separable and linear

(iv) Neither separable nor linear

2. Answer the following questions ***(any five)***: 2×5=10

(a) Consider the function:

$$f(x)=\begin{cases}\frac{x^2-9}{x-3}, & if\ x\neq 3\\ 6, & if\ x=3\end{cases}$$

Check whether $f(x)$ is continuous at $x=3$ or not.

Contd.

(b) Show that the position vector $\vec{r}$ is irrotational.

(c) Find the unit vertor perpendicular to each of the vectors

$\vec{A} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{B} = 3\hat{i} + 4\hat{j} - \hat{k}$

(d) If $f(x,y) = 2x^3 - x^2y^6$, evaluate

$\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

(e) State Bayes' theorem on conditional probability of events.

(f) Find a suitable integrating factor to make the inexact differential equation $x^2ydx - (x^3 + y^3)dy = 0$ to an exact differential equation.

(g) Evaluate $\int_{x=0}^{3} \int_{y=1}^{2} (4xy - y^2)dxdy$.

3. Answer the following questions ***(any five)*** : 5×5=25

(a) (i) Find a unit outward normal drawn to the surface $z = x^2 + y^2$ at the point (1,2,5). 2

(ii) What is Poisson's distribution? Calculate the mean of Poisson's distribution. 1+2

(b) Find the general solution of the differential equation $xdy - (3y + x^4)dx = 0$.

(c) How the spherical coordinates are related to cartesian coordinates? Show that the spherical coordinate system is an orthogonal curvilinear coordinate system. 2+3=5

(d) Show that $\vec{\nabla}(r^n\vec{r}) = (n+3)r^n$,

where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

(e) State Green's theorem. Show that the area bounded by a simple closed curve is given by $\frac{1}{2}\oint_c (xdy - ydx)$.

Hence find the area of the ellipse $x = a\cos\theta,\ y = b\sin\theta$. 1+2+2=5

(f) (i) If $\vec{F} = 2z\hat{i} - x\hat{j} + y\hat{k}$, evaluate $\iiint_V \vec{F}dV$, where V is the region bounded by the surfaces $x = 0, y = 0, x = 2, y = 4, z = x^2$ and $z = 2$. 3

(ii) Prove that $\vec{A}.(\vec{A}\times\vec{B})=0$. 2

(g) (i) Solve the equation $y''+2y'-3y=0$ with $y(0)=1$, $y'(0)=5$. 3

(ii) Using the help of Wronskian, show that the functions x and $\sin x$ are linearly independent. 2

(h) What is an exact differential equation? Write down the necessary and sufficient condition to be an exact differential equation. Show that the differential equation $(x-xy^2)dx+(8y-x^2y)dy=0$ is exact and hence solve it. 1+1+3=5

4. Answer the following question ***(Any One)***: 10

(a) (i) Derive the expression for gradient of a scalar in curvilinear coordinates and convert it to cylindrical coordinates. 5

(ii) Find the particular integral of

$$\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=e^x\sin x.$$ 5

(b) (i) Evaluate $\iint_S \vec{A}.\hat{n}ds$, where $\vec{A}= yz\hat{i}+zx\hat{j}+xy\hat{k}$ and S is the surface of the sphere $x^2+y^2+z^2=a^2$ in the first octant. 7

(ii) State Stoke's theorem. Using Stoke's theorem, prove that $\oint_C \vec{r}.\overline{dr}=0$. 1+2