

2023

(Held in 2024)

MATHEMATICS

Paper : MATMAJ1014

(**Foundation of Mathematics**)

Full Marks : 50

Pass Marks : 20

Time : 2 hours

*The figures in the margin indicate full marks
for the questions*

1. Choose the correct answer from the following : 1×5=5

(a) If α , β and γ are the roots of the equation $x^3 + px^2 + r = 0$, then the value of $\Sigma \alpha \beta$ is

(i) 0

(ii) 1

(iii) $-p$

(iv) $-r$

(b) Let $X = \mathbb{Z}^*$ be the set of non-zero integers. We define a relation R on X by setting mRn if and only if m divides n . Then R is

- (i) reflexive and symmetric
- (ii) transitive and symmetric
- (iii) equivalence relation
- (iv) reflexive, transitive but not symmetric

(c) Let $f: [2, \infty) \rightarrow \mathbb{R}$ be a function defined by $f(x) = x^2 - 4x + 5$. Then the range of f is

- (i) $(-\infty, \infty)$
- (ii) $[1, \infty)$
- (iii) $[2, \infty)$
- (iv) $[3, \infty)$

(d) If A and B are $n \times n$ invertible matrices, then so is AB , and then $(AB)^{-1}$ is equal to

- (i) $A^{-1}B^{-1}$
- (ii) BA
- (iii) $B^{-1}A^{-1}$
- (iv) AB

(e) If a and b are integers such that $a|b$, then $\gcd(a, b)$ is

- (i) a
- (ii) b
- (iii) $|a|$
- (iv) $|b|$

2. Answer any five of the following questions :
2×5=10

(a) Without actual division, find the remainder when $x^4 - 3x^2 - x - 6$ is divided by $x + 3$.

(b) Let $X = \{a, b, c\}$ and $Y = \{1, 2, 3\}$. Consider the subset R of $X \times Y$ as

$$R = \{(a, 1), (b, 1), (a, 3), (b, 2)\}$$

(i) In this relation, is a related to 2?

(ii) What are the elements in X to which 3 is related under R ?

(c) Find a quartic equation having the two double roots 3 and -3 .

(d) Consider the functions $f: \mathbb{Z} \rightarrow \mathbb{Z}$ and $g: \mathbb{N} \rightarrow \mathbb{N}$ defined by $f(x) = g(x) = x^2$. Is $f = g$? Give answer with proper reasoning.

(e) Given $\bar{u} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ and $\bar{v} = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$, find $4\bar{u} + (-3)\bar{v}$.

(f) Suppose a , b and c are integers such that $c|a$ and $c|b$, then show that $c|(xa + yb)$ for some integers x and y .

(g) Find the inverse of

$$A = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$$

by row operations.

3. Answer any *five* of the following questions :

$$5 \times 5 = 25$$

(a) State and prove fundamental theorem of arithmetic.

(b) If $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ are bijections, then prove that $g \circ f$ is a bijection and $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.

(c) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be given by $f(x) = x^2$. Then verify that $f([-4, 3]) = [0, 16]$.

(d) Prove that one root of

$$x^3 + px^2 + qx + r = 0$$

is the negative of another root if and only if $pq = r$.

(e) If an algebraic equation with real coefficients has the root $a + bi$, where a and b are real and $b \neq 0$, prove that it has also the root $a - bi$.

(f) Use invertible matrix theorem to decide if A is invertible, where

$$A = \begin{bmatrix} 1 & 0 & -2 \\ 3 & 1 & 2 \\ -5 & -1 & 9 \end{bmatrix}$$

(g) If A is an invertible matrix, then show that A^{-1} is invertible and $(A^{-1})^{-1} = A$.

(h) Find all the rational roots of

$$2x^3 - 7x^2 + 10x - 6 = 0$$

4. Answer any *one* of the following questions : 10

(a) (i) Solve the following equation by Cardan's method : 5

$$x^3 - 9x + 28 = 0$$

(ii) Apply elementary row operations to transform the following matrix first into echelon form and then into reduced echelon form : 5

$$\begin{bmatrix} 0 & 3 & -6 & 6 & 4 & -5 \\ 3 & -7 & 8 & -5 & 8 & 9 \\ 3 & -9 & 12 & -9 & 6 & 15 \end{bmatrix}$$

- (b) (i) Define equivalence relation on a non-empty set X . On \mathbb{R}^2 , we define $(x_1, y_1) \sim (x_2, y_2)$ iff $x_1^2 + y_1^2 = x_2^2 + y_2^2$. Verify that \sim is an equivalence relation. What is the equivalence class of a point (a, b) ? Interpret the equivalence classes geometrically.
2+3+2+1=8
- (ii) Find the greatest common divisor of 630 and 196. 2

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