Total number of printed pages–7

63 (FY)SEM–2/MIN2/MATMIN1024

2024

MATHEMATICS

Paper : MATMIN1024

(Integral Calculus and Differential Equations)

Full Marks : 70

Pass Marks : 28

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : 1×8=8

   (i) The Clairaut's equation takes the following form :

   (a) $y = p(x) + f(p)$

   (b) $y' = p(x) + f(p)$

   (c) $y = p(x) + f'(p)$

   (d) $y = p'(x) + f(p)$

Contd.

(ii) $\int \frac{1}{x^2+a^2}dx$ is equal to

(a) $\frac{1}{a^2}\tan^{-1}\frac{x}{a}+c$

(b) $\frac{1}{a}\tan^{-1}\frac{x}{a}+c$

(c) $\frac{1}{a}\sec^{-1}\frac{x}{a}+c$

(d) $\frac{1}{a}\sin^{-1}\frac{x}{a}+c$

(iii) $\int_a^b e^{kx}dx$ is equal to

(a) $\frac{1}{a-b}\left(e^{kb}-e^{ka}\right)$

(b) $\frac{1}{a-b}\left(e^{ka}-e^{kb}\right)$

(c) $\frac{1}{k}\left(e^{ka}-e^{kb}\right)$

(d) $\frac{1}{k}\left(e^{kb}-e^{ka}\right)$

(iv) $\int_0^{na} f(x)dx = n\int_0^a f(x)dx$, if

(a) $f(x) = f(a-x)$
(b) $f(x) = f(a+x)$
(c) $f(a-x) = f(a+x)$
(d) $f(x) = a\,f(x)$

(v) The order and degree of partial differential equation $\left(\frac{\partial z}{\partial x}\right)^2+\frac{\partial^3 z}{\partial y^3}=2x\frac{\partial z}{\partial x}$ are

(a) order 1, degree 2
(b) order 2, degree 2
(c) order 1, degree 1
(d) order 3, degree 1

(vi) The differential equation $Mdx+Ndy=0$ where M and N are functions of x and y will be exact, if

(a) $\frac{\partial M}{\partial x}=\frac{\partial N}{\partial y}$

(b) $\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$

(c) $\frac{\partial M}{\partial x}\neq\frac{\partial N}{\partial y}$

(d) $\frac{\partial M}{\partial y}\neq\frac{\partial N}{\partial x}$

Contd.

(vii) The solution of the differential equation $(1 + y^2)dx + (1 + x^2)dy = 0$ is

(a) $\tan x + \tan y = c$

(b) $\tan^{-1} x + \tan^{-1} y = c$

(c) $\sin^{-1} x + \sin^{-1} y = c$

(d) $\cos^{-1} x + \cos^{-1} y = c$

(viii) The value of $\int_{-\pi/2}^{\pi/2} \sin^7 x dx$ is

(a) 1

(b) −1

(c) 0

(d) 2

2. Answer the following questions **(any six)** : 2×6=12

(i) Test the linear independence of the following functions $1 + x$, $1 + 2x$, x^2.

(ii) Show that $y = A \cos x + B \sin x$ is a solution of the differential equation $\frac{d^2y}{dx^2} + y = 0$.

(iii) Evaluate : $\int xe^{3x}dx$.

(iv) Find the integrating factor of $\frac{dy}{dx} + \left(\frac{2x+1}{x}\right)y = e^{-2x}$.

(v) Find the value of $\int_0^{\pi/2} x \sin x dx$.

(vi) Form a partial differential equation by eliminating a and b from $z = (x + a)(y + b)$.

(vii) Solve : $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 8y = 0$.

(viii) If $\int_0^a 3x^2 dx = 8$, find the value of a.

3. Answer the following questions **(any six)** : 5×6=30

(i) Find the complete and singular solution of $y = px + p - p^2$.

(ii) Integrate : $\int \frac{e^x}{e^{2x} + 2e^x + 5} dx$.

(iii) Solve : $\frac{dy}{dx} + \frac{1}{x}y = y^2$.

(iv) Solve : $(D-2)^2 y = 6\, e^{2x}$, where $D = \frac{d}{dx}$.

(v) Show that $\int_0^{\pi/2} \cos^6 x dx = \frac{5}{32}\pi$.

(vi) Find the general solution of the differential equation

$$x^2 \frac{\partial z}{\partial x} + y^2 \frac{\partial z}{\partial y} = (x+y) z.$$

(vii) Solve the equation :

$$(3x^2 + 4xy)dx + (2x^2 + 2y)dy = 0.$$

(viii) Show that e^{2x} and e^{3x} are linearly independent solutions of $y'' - 5y' + 6y = 0$.

(ix) Solve by the method of variation of parameters $\frac{d^2 y}{dx^2} + y = \sec x$.

4. Answer the following questions ***(any two)*** : 10×2=20

(a) *(i)* Obtain the reduction formula for $\int \tan^n x dx$.

(ii) If $I_n = \int_0^{\pi/4} \tan^2 x dx$, then show that $n(I_{n+1} + I_{n-1}) = 1$.

6+4=10

(b) Find the solution of the differential equation $\frac{d^2 y}{dx^2} - 2\frac{dy}{dx} - 3y = 2e^x - 10 \sin x$.

(c) *(i)* Solve : $\frac{d^2 y}{dx^2} + y = x^3$.

(ii) Eliminate arbitrary function f and F from $y = f(x - at) + 7(x + at)$.

6+4=10

(d) *(i)* Obtain the reduction formula for $\int \sin^n x dx$.

(ii) Evaluate : $\int_0^{\pi/2} \sin^3 x \cos^5 x dx$.

5+5=10