*Total number of printed pages–11*

**63/1 (SEM–6) DSE3/MATHE6036**

**2024**

# MATHEMATICS

Paper : MATHE6036

***( Theory of Equations )***

*Full Marks : 80*

*Pass Marks : 32*

Time : Three hours

***The figures in the margin indicate full marks for the questions.***

1. Choose the correct answer : ***(any six)*** $1 \times 6 = 6$

   *(a)* An equation of 5th degree is known as

   *(i)* Quadratic Equation

   *(ii)* Cubic Equation

   *(iii)* Quintic Equation

   *(iv)* Quartic Equation

*Contd.*

(b) Every equation of an odd degree has at least ______ of a sign opposite to that of its last term.

(i) one real root

(ii) two real roots

(iii) three real roots

(iv) four real roots

(c) Roots of the cubic equation $x^3 - 1 = 0$ are

(i) one real and one imaginary

(ii) one real and two imaginary

(iii) two real and one imaginary

(iv) all real

(d) The equation $x^8 - 10x^3 - x - 4 = 0$ has at least

(i) 2 imaginary roots

(ii) 4 imaginary roots

(iii) 6 imaginary roots

(iv) 8 imaginary roots



(e) If the roots of an equation be all positive, the co-efficient (including the co-efficient of highest power) should be

(i) all positive

(ii) all negative

(iii) all positive or negative

(iv) alternately positive and negative

(f) If $\alpha, \beta, \gamma$ be the roots of the equation $x^3 + px^2 + qx + r = 0$, then the value of $\sum \frac{1}{\alpha}$ is

(i) $\frac{q}{r}$

(ii) $-\frac{q}{r}$

(iii) $\frac{r}{q}$

(iv) $-\frac{r}{q}$

*(g)* The minimum value of $f(x) = 2x^2 + x - 6$ is

*(i)* $-\frac{49}{8}$

*(ii)* −49

*(iii)* 8

*(iv)* 49

*(h)* Between two consecutive real roots *a* and *b* of the equation *f*(*x*) = 0, there lies ______ real root of the equation $f'(x) = 0$.

*(i)* no

*(ii)* exactly one

*(iii)* at least one

*(iv)* at most one



*(i)* If the sum of two roots of the equation $x^3 + 6x^2 - 3x - 18 = 0$ is zero, then its roots are

*(i)* $-\sqrt{2}$, $\sqrt{2}$, 6

*(ii)* $-\sqrt{2}$, $\sqrt{2}$, −6

*(iii)* $-\sqrt{3}$, $\sqrt{3}$, −6

*(iv)* $-\sqrt{3}$, $\sqrt{3}$, 6

*(j)* A necessary condition for $f(x) = 0$ to be a reciprocal equation is that

*(i)* $f(0) = 0$

*(ii)* $f(0) \neq 0$

*(iii)* $f(0) = 1$

*(iv)* $f(0) \neq 1$

2. Answer the following questions : ***(any five)*** 2×5=10

*(a)* Find the quotient and remainder when $x^3+5x^2+3x+2$ is divided by $(x-1)$.

*(b)* If $\alpha, \beta, \gamma$ are the roots of the equation $x^3+px^2+qx+r=0$, find the value of $\alpha^2+\beta^2+\gamma^2$.

*(c)* Solve $x^3+3x^2-10x-24=0$ if one root is 3.

*(d)* Find the equation whose roots are the roots of the equation

$$x^3+3x^2-8x+1=0$$

each diminished by 4.

*(e)* If $\alpha_1, \alpha_2, \alpha_3, \ldots\ldots, \alpha_n$ are the roots of the equation

$$x^n-p_1x^{n-1}+p_2x^{n-2}-p_3x^{n-3}+\ldots\ldots+(-1)^n p_n=0,$$

find the value of $(1+\alpha_1)(1+\alpha_2)\ldots\ldots(1+\alpha_n)$.



*(f)* If $\alpha$ be any root of $x^n-1=0$, then prove that $\alpha^m$ is also a root of the equation, where $m$ is any integer.

*(g)* Find the common roots of $x^4-1=0$ and $x^6-1=0$.

3. Answer the following questions : ***(any six)*** 5×6=30

*(a)* Prove that every equation of $n$ dimensions has $n$ roots and no more.

*(b)* Find the equation where roots are the squares of the roots of the equation $2x^3-3x^2+4x-5=0$.

*(c)* Find the condition that the cubic equation $x^3+px^2+qx+r=0$ should have two roots $\alpha, \beta$ connected by the relation $\alpha\beta+1=0$.

*(d)* Find the equation whose roots are the reciprocals of the roots of $x^4-3x^3+7x^2+5x-2=0$.

*(e)* Solve the equation $x^3-9x+28=0$.

(f) If $\alpha, \beta, \gamma$ be the roots of the cubic $x^3 - px^2 + qx - r = 0$ form the equation whose roots are $\beta\gamma + \frac{1}{\alpha}$, $\gamma\alpha + \frac{1}{\beta}$, $\alpha\beta + \frac{1}{\gamma}$.

(g) Show that the equation $4(x^2 - x + 1)^3 - 27x^2(x-1)^2 = 0$ is reciprocal, and solve it.

(h) Find the multiple roots and the remaining factor of the equation $x^5 - 10x^2 + 15x - 6 = 0$.

(i) Find the sum of the fourth powers of the roots of the equation $x^3 - x - 1 = 0$.

(j) Determine $k$ and solve the following equation if its roots are in the harmonic progression :

$$8x^3 - 12x^2 - kx + 3 = 0.$$

4. Answer the following questions : ***(any two)*** 10×2=20

(a) If the equation $x^3 + px^2 + qx + r = 0$ has a root $\alpha + i\alpha$, where $p$, $q$, $r$ and $\alpha$ are reals, then prove that

$$(p^2 - 2q)(q^2 - 2pr) = r^2$$



(b) (i) Apply Descartes' rule of signs to show that the equation $x^4 + 2x^3 + 3x - 1 = 0$ has exactly one pair of complex roots. 5

(ii) Transform the equation

$$x^4 + 8x^3 + x - 5 = 0$$

into one in which the second term is wanting. 5

(c) Find the condition that the cubic $ax^3 + 3bx^2 + 3cx + d$ may be capable of being written under the form $l(x-\alpha_1)^3 + m(x-\beta_1)^3 + n(x-\gamma_1)^3$ where $\alpha_1, \beta_1, \gamma_1$ are the roots of the cubic $a_1x^3 + 3b_1x^2 + 3c_1x + d_1 = 0$.

(d) Apply Sturm's theorem to the analysis of the equation

$$x^4 - 4x^3 + 7x^2 - 6x - 4 = 0.$$

5. Answer the following questions : ***(any one)***

1×14=14

*(a)* *(i)* Prove that every equation of an even degree, whose last term is negative, has at least two real roots — one positive and the other negative. 8

*(ii)* Show that the equation

$$\frac{A^2}{x-a}+\frac{B^2}{x-b}+\frac{C^2}{x-c}+\ldots\ldots+\frac{L^2}{x-l}=x-m$$

where $a$, $b$, $c$, ....., $l$, $m$ are numbers all different from one another, cannot have an imaginary root. 6

*(b)* *(i)* Show all the roots of the equation

$$x^n+p_1x^{n-1}+p_2x^{n-2}+\ldots\ldots+p_{n-1}x+p_n=0$$

can be obtained when they are in arithmetical progression. 10



*(ii)* Find in terms of $p$, $q$, $r$ the value of the symmetric function

$$\frac{\beta^2+\gamma^2}{\beta\gamma}+\frac{\gamma^2+\alpha^2}{\gamma\alpha}+\frac{\alpha^2+\beta^2}{\alpha\beta},$$ where

$\alpha,\beta,\gamma$ are the roots of the equation $x^3+px^2+qx+r=0$. 4

*(c)* *(i)* If $m$ and $n$ be prime to each other, then prove that the equations $x^m-1=0$ and $x^n-1=0$ have no common root except unity. 6

*(ii)* Find the integral roots of

$$3x^4-23x^3+35x^2+31x-30=0$$

8

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