Total number of printed pages–10

63/1 (SEM–5) DSE1A/MATRE5016

2024

MATHEMATICS

Paper : MATRE5016

(Mechanics)

Full Marks : 80

Pass Marks : 32

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Choose the correct answer : ***(any six)*** 1×6=6

   (a) A particle in equilibrium under the action of severel forces satisfies–

   (i) the sum of forces is constant

   (ii) the forces are perpendicular to each other

   (iii) the sum of forces is zero

   (iv) the resultant force is not zero

(b) The coefficient of static friction depends on–

(i) the nature of the surfaces in contact

(ii) the area of contact

(iii) the speed of the moving object

(iv) the temperature of the environment

(c) The centre of gravity of an object is–

(i) the piont where the entire weight of the object acts

(ii) the point where the mass of the object is zero

(iii) the point where the volume of the object is maximum

(iv) the point where density is minimum

(d) The total work done by a conservative force over a closed path is-

(i) always zero

(ii) always positive

(iii) always negative

(iv) dependent on the shape of the path

(e) In Simple Harmonic Motion, the restoring force is proportional to–

(i) the velocity

(ii) the displacement from the equilibrium position

(iii) the acceleration

(iv) the mass of the object

(f) The time period of a simple pendulum is proportional to–

(i) the mass of the pendulum bob

(ii) the square root of its length

(iii) the amplitude of oscillation

(iv) the gravitational force acting on the pendultum

(g) In projectile motion, the horizontal range is maximum when the angle of projection is–

(i) 30°

(ii) 45°

(iii) 60°

(iv) 90°

(h) The velocity of a particle in curvilinear motion has two components, which are–

(i) radial and normal

(ii) tangential and normal

(iii) radial and transverse

(iv) radial and longitudinal

(i) Work is defined as–

(i) force divided by distance

(ii) force times distance

(iii) force times velocity

(iv) velocity divided by time

(j) Newton's First Law is also know as :

(i) Law of force

(ii) Law of action and reaction

(iii) Law of acceleration

(iv) Law of ineria

2. Answer the following questions : ***(any five)*** 2×5=10

(a) A particle moves along a straight line such that its displacement x, from a point on the line at time t, is given by. $x = t^3 - 9t^2 + 24t + 6$.

Determine :

(i) the instant when the acceleration becomes zero

(ii) the position of the particle at that instant.

(b) State the necessary and sufficient conditions for equilibrium of rigid bodies in two-dimension.

(c) The position vector and force are $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $20\hat{i} - 260\hat{j} + 320\hat{k}$ respectively. Find the moment of the force about the origin and also find the scalar quantity of the moment..

(d) What is the relationship between radial and transverse components of velocity?

(e) State Newton's Third Law of Motion with an example.

(f) Define centre of gravity and give *one* example.

(g) Explain the difference between static and kinetic friction.

3. Answer the following questions : ***(any six)*** 5×6=30

(a) Show that a system of coplanar forces acting upon a rigid body will keep it in equilibrium, if the algebraic sum of the moment of the forces about each of three non-collinear points vanish.

(b) Two rough particles connected by a light string rest on an in clined plane. If their weights and corresponding coefficients of friction are w_1, w_2 and μ_1, μ_2 respectively. Show that greatest inclination of the plane for equilibrium is $\tan^{-1}\left[(\mu_1 w_1 + \mu_2 w_2)/(w_1 + w_2)\right]$

(c) Find the centre of gravity of the sector of a circle subtending an angle 2α at the centre of the circle.

(d) A particle moves in a straight line with constant acceleration and its distances from the origin O on the line at times t_1, t_2, t_3 and x_1, x_2, x_3 respectively. Show that if t_1, t_2, t_3 form an A.P whose common difference is d and x_1, x_2, x_3 are G.P., then the acceleration is $\left(\sqrt{x_1} - \sqrt{x_3}\right)^2 / d^2$.

(e) The maximum velocity of a body moving with Simple Harmonic Motion is $2\,ft/sec.$ and its period is $\frac{1}{5} sec.$ What is its amplitude?

(f) State Newton's 2nd Law of Motion. Establish the relation $P = mf$. (where P, m, f are usual meaning)

(g) A particle projected from a point O with velocity v in a direction making an angle of elevation α with the horizontal direction. Show that the kinetic energy of the particle on the projectile at $P(x, y)$ is given by $K = mgH - mgy$, where $H = v^2/2g$.

(h) If R be the horizontal range and h the greatest height of a projectile, prove that the initial velocity is $\left[2g\left(h + \frac{R^2}{16h}\right)\right]^{1/2}$.

(i) A body of mass $10kg$ is pushed on a surface with a force of $50N$. The coefficient of kinetic friction is 0.3. Calculate the acceleration of a body.

(j) What is a simple pendulum ? Derive an expression for its time period.

4. Answer the following questions : ***(any two)*** 10×2=20

(a) Three forces P, Q, R act along the sides of a triangle formed by the line $x+y=1$, $y-x=1$ and $y=2$. Find the equation of the line of action of the resultant.

(b) A body is placed on a rough inclined plane, which makes an angle greater than angle of friction with the horizontal, and is prevented from moving by a force acting parallel to the line of greatest slope. Find the limits of this force.

(c) The trajectory of a projectile in vaccum is the parabola, $y = x\tan\alpha - \frac{1}{2}g\frac{x^2}{u^2\cos^2\alpha}$ where u is the initial velocity of the projectile, α the angle of pojection. Find the Latus-Rectum, Vertex. Focus and Directrix of the trajectory.

(d) Explain the concept of tangential and normal components of a acceleration for a particle moving along a curve.

5. Answer the following questions : ***(any one)*** 14×1=14

(a) A uniform rod of length 'a' hangs against a smooth vertical wall being supported by means of a string of length 'l', tied to one end of the rod, the other end of the string attached to a print in the wall; show that the rod can rest inclined to the wall at an angle θ given by $\cos^2\theta = (l^2 - a^2)/3a^2$,

What are the limits of the ratio of $a:l$ that the equilibrium may be possible? 10+4=14

(b) (i) Find the C.G. of the region bounded by the parabola $x^2 = 4ay$, x-axis and the ordinate $x=b$. 7

(ii) Find the least force 'P' required to pull a body of weight 'w' on a rough horizontal plane. The force 'P' makes an angle α with the horizon and the λ the angle of friction. 7

(c) (i) If t be regarded as a function of velocity v, Prove that the rate of decrease of acceleration is given by $f^3(d^2t/dv^2)$, f being the acceleration. 7

Contd.

(ii) If a pendulum of length makes n complete oscillations in s given time. Show that if the length be changed to $l+l'$, the number of oscillations lost is $nl'/(2l)$. 7
