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63/1 (SEM–5) DSE2/MATHE5026

2024

MATHEMATICS

Paper : MATHE5026

(Probability and Statistics)

Full Marks : 80

Pass Marks : 32

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Choose the correct answer: ***(any six)*** $1\times6=6$

(a) The probability of a sure event is

(i) 0

(ii) 1

(iii) 02

(iv) 03

Contd.

(b) The events E and F are mutually exclusive and $P(E)=0.4$, $P(F)=0.2$. Then $P(E \cup F)$ is

(i) 0.08

(ii) 0

(iii) 0.6

(iv) 0.9

(c) E and F are two events which have no point in common, then the events E and F are

(i) Independent

(ii) Dependent

(iii) Mutually exclusive

(iv) Exhaustive

(d) Which of the following is a discrete distribution?

(i) Binomial distribution

(ii) Uniform distribution

(iii) Normal distribution

(iv) Exponential distribution

(e) The limiting relative frequency approach of probability is known as

(i) Statistical probability

(ii) Classical probability

(iii) Mathematical probability

(iv) Conditional probability

(f) For which type of probability distribution are the mean and variance equal ?

(i) Poisson distribution

(ii) Geometric distribution

(iii) Pascal distribution

(iv) Uniform distribution

(g) Let X be a random variable. Then for

$f(x) = \lambda e^{-2x}, x > 0$

$= 0$ otherwise

to be density function, λ must be equal to

(i) 2

(ii) ½

(iii) 0

(iv) 1

(h) The value of λ for which the probability mass function

x	-1	0	1
$P(x)$	0.4	0.6	λ

admissible is

(i) 0

(ii) 1

(iii) 2

(iv) −1

(i) The covariance of two independent variables is

(i) 1

(ii) 2

(iii) 3

(iv) 0

(j) The moment generating function of a discrete random variable X is given by

(i) $M_X(t) = \sum_x e^{tx}$

(ii) $M_{X(t)} = \sum_x e^t f(x)$

(iii) $M_X(t) = \sum_x e^{tx} f(x)$

(iv) $M_X(t) = \sum_x f(x)$

2. Answer the following questions : ***(any five)*** 2×5=10

(a) Let E and F be two events with $P(E) = \frac{1}{3}$, $P(F) = \frac{1}{4}$ and $P(E \cap F) = \frac{1}{2}$. Find $P(E/F)$.

(b) Define joint probability mass function.

(c) Prove that

$$cov(ax, by) = ab\ cov(x, y)$$

(d) Define state-transition matrix.

(e) Prove that the mean of a binomial distribution is always greater than the variance.

(f) A random variable X has density function given by

$$f(x) = 2e^{-2x}, x \geq 0$$
$$= 0 \quad , x < 0$$

Find the moment generating function.

(g) If $B \subset A$, then prove that

$P(A \cap \bar{B}) = P(A) - P(B)$, where $\bar{B}$ is complement of B.

3. Answer ***any six*** of the following questions : 5×6=30

(a) Mother, father and son line up at random for a family picture. Find $P(A/B)$ if A and B are defined as follows:

A = Son on one end, B = Father in the middle.

(b) Let X be a continuous random variable with p.d.f. is given by

$$
\begin{aligned}
f(x) &= ax &&, 0 \le x \le 1 \\
&= a &&, 1 \le x \le 2 \\
&= -ax + 3a &&, 2 \le x \le 3 \\
&= 0 &&, 3 < x.
\end{aligned}
$$

(i) Find the value of a.

(ii) Find $P(x \le 1.5)$.

(c) State and prove the total theorem of probability.

(d) Define covariance between two random variables X and Y, and prove that it is independent of change of the origin.

(e) Let $X_1, X_2, X_3, \ldots, X_n$ be mutually independent random variables (discrete or continuous), each having finite mean μ and variance σ^2. Then if $S_n = X_1 + X_2 + X_3 + \ldots + X_n$, $(n = 1, 2, 3, \ldots)$, prove that

$$\underset{n \to \infty}{Lt} P\left(\left|\frac{S_n}{n} - \mu\right| \ge \varepsilon\right) = 0$$

(f) A random variable has the following probability distribution

x	0	1	2	3
$P(x)$	0.1	0.3	0.4	0.2

Find—*(i)* $E(X)$ *(ii)* $var(X)$.

(g) Derive Poisson distribution as a limiting case of binomial distribution.

(h) The probability density function of a continuous bivariate distribution is given by the joint density function

$f(x,y) = x + y,\ 0 < x < 1,\ 0 < y < 1$

$= 0$, elsewhere

Find $E(X)$, $E(Y)$, $var(X)$, $var(Y)$ and $E(XY)$.

(i) If the random variable X is normalty distributed with mean μ and variance σ^2, show that the mean of the variate

$$z = \frac{X - \mu}{\sigma}$$ is always zero.

(j) Explain Markov Chain.

4. Answer the following questions : ***(any two)*** 10×2=20

(a) State and prove central limit theorem for independent and identically distributed random variables with finite variance. 2+8=10

(b) (i) Define characteristic function and prove that the characteristic function $\phi_X(t)$ is bounded by 1. 5

(ii) Derive the mean and variance of geometric distribution. 5

(c) (i) For the distribution

$$P(X = x) = 2^{-x},\ x = 1, 2, 3, \ldots..$$

prove that Clebyshev's inequality gives $P[|X-2| \leq 2] > \frac{1}{2}$ while the actual probability is 15/16. 5

(ii) Prove that, for any random variable X, $var(aX+b) = a^2 var(X)$, where a, b are real constants. 5

(d) (i) Prove that

$$var(X) = E[var(X|Y)] + var[E(X|Y)].$$ 5

(ii) The joint probability density function of X and Y is given by

$$f(x) = \frac{2}{3}(x+2y),\ 0<x<1,\ 0<y<1$$
$$= 0, \qquad \text{elsewhere}$$

Find the conditional mean and conditional variance of X given $Y = \frac{1}{2}$. 5

5. Answer ***any one*** of the following questions : 14×1=14

(a) State and prove Chebyshev's theorem and explain the law of large number. 2+7+5=14

(b) (i) Let X be a random variable and $aX+b$ be its linear function, then prove that $M_{aX+b}(t) = e^{bt} M_X(at)$. 5

(ii) The school of international studies for population found out through its survey, that the mobility of the population of a state to a village, town and city is in the following transition matrix.

		To Village	To Town	To City
From	Village	0.5	0.3	0.2
	Town	0.1	0.7	0.2
	City	0.1	0.4	0.5

What will be the proportion of population in village, town and city after two years, given that the present population has proportions of 0.7, 0.2 and 0.1 in the village, town and city respectively. 5

(iii) State and prove Schwarz's inequality. 4

(c) What ins co-relation coefficient ? Define positive and negative co-relation.

Prove that co-relation coefficient does not depend on the change of origin and scale.

If $cov(X,Y)=2$, $\sigma_x=1$ and $\sigma_y=2$, then find co-relation coefficient. 2+2+2+6+2=14
