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63/1 (SEM–1) CC2/MATHC 1026

2024

MATHEMATICS

Paper : MATHC 1026

(Algebra)

Full Marks : 80

Pass Marks : 32

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Choose the correct answer from the following : ***(any six)*** 1×6=6

   (a) The polar representation of the complex number $2i$ is

      (i) $2(\cos\pi + i\sin\pi)$

      (ii) $2(\cos\pi - i\sin\pi)$

      (iii) $2(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2})$

      (iv) $2(\cos\frac{\pi}{2} - i\sin\frac{\pi}{2})$

(b) The greatest common divisor of -24 and 6 is

(i) -6

(ii) 6

(iii) -24

(iv) 3

(c) If $n(A)=4$, then the number of one-one functions from A to A is

(i) 12

(ii) 16

(iii) 20

(iv) 24

(d) If x is a natural number, then the cordinality of the set $\{x : x^2 - 9 = 0\}$ is

(i) 0

(ii) 1

(iii) 2

(iv) 4

(e) A scalar λ is an eigenvalue of an $n \times n$ matrix A, if and only if

(i) det $(A+\lambda I)=\lambda$

(ii) det $(A-\lambda I)=0$

(iii) det $(A-\lambda I)=\lambda$

(iv) det $(A+\lambda I)=0$

(f) If L is the l.c.m. of two non-zero integers a and b and c is any integer such that $a|c$ and $b|c$, then

(i) $c \le L$

(ii) $c \ge L$

(iii) $c = L$

(iv) $c > L$

(g) The rank of a 5×6 matrix can be at most

(i) 5

(ii) 6

(iii) 4

(iv) 3

(h) The dimension of the vector space $\mathbb{R}^6(\mathbb{R})$ is

(i) 4

(ii) 5

(iii) 6

(iv) 0

(i) Well-ordering principle states that any non-empty set of

(i) positive real numbers has a smallest element

(ii) negative real numbers has a smallest element

(iii) integers has a smallest element

(iv) natural numbers has a smallest element

(j) Let A be the set of all employees of Bodoland University. Then the relation $R = \{(x,y) : x \text{ is a son of } y\},\ x, y \in A$, is

(i) reflexive

(ii) symmetric

(iii) both reflexive and symmetric

(iv) neither reflexive nor symmetric

2. Answer ***any five*** of the following questions : 2×5=10

(a) Evaluate : $(1+i)^{1000}$

(b) Solve : $3x \equiv 1 \pmod 5$

(c) Find the eigenvalues of the following matrix :

$$\begin{bmatrix} 3 & 6 & -8 \\ 0 & 0 & 6 \\ 0 & 0 & 2 \end{bmatrix}$$

(d) Check whether the relation R on $\mathbb{Z}$ defined by $R = \{(a, b) : a - b \text{ is divisible by } 3,\ a, b \in \mathbb{Z}\}$ is an equivalence relation.

(e) Suppose a, b, x are integers such that $a \mid bx$. If a and b are relatively prime then show that a divides x.

(f) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that $T(x,y) = (x+y,\ 4x+5y)$. Find $\alpha \in \mathbb{R}^2$ such that $T(\alpha) = (3, 8)$.

(g) Let $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ be defined by $f(p,q) = p + q;\ p, q \in \mathbb{Z}$. Examine whether f is one-one and onto.

3. Answer ***any six*** of the following questions : 5×6=30

(a) Find the nth roots of unity.

(b) Prove that the composition of functions is an associative operation.

(c) Find the cardinality of $\mathbb{N} \times \mathbb{N}$, where $\mathbb{N}$ is the set of all natural numbers.

(d) If a natural number $n > 1$ is not prime, then prove that n is divisible by some prime number $p \le \sqrt{n}$.

(e) Let $v_1 = \begin{bmatrix} 3 \\ 0 \\ -6 \end{bmatrix},\ v_2 = \begin{bmatrix} -4 \\ 1 \\ 7 \end{bmatrix},\ v_3 = \begin{bmatrix} -2 \\ 1 \\ 5 \end{bmatrix}$.

Examine if $\{v_1, v_2, v_3\}$ is a basis for the vector space $\mathbb{R}^3(\mathbb{R})$.

(f) Prove that the eigenvalues of triangular matrix are the entries on its main diagonal.

(g) Prove that there are infinitely many primes.

(h) If a, $a+1$, $a+2$ are three consecutive integers, then prove that one of them is divisible by 3.

(i) Prove that $\dfrac{(1-i)^{10}(\sqrt{3}+i)^{5}}{\left(-1-i\sqrt{3}\right)^{10}}=-1$.

(j) For any natural number $n > 1$, prove that there exists a prime p such that $p \mid n$.

4. Answer ***any two*** of the following : 10×2=20

(a) (i) Solve the following system of linear equations : 6

$$3x-2y+5z=7$$
$$7x+4y-8z=3$$
$$5x-3y-4z=-12$$

(ii) If $f: A \to B$ and $g: B \to C$ be two invertible functions, then show that $g \circ f$ is invertible. 4

(b) (i) If a vector space V has a basis $S=\{v_1, v_2, v_3, \ldots, v_n\}$, then prove that any set in V containing more than n vectors must be linearly dependent. 8

(ii) Prove that the subset $\{(2,3)\}$ of the vector space $\mathbb{R}^2(\mathbb{R})$ is linearly independent. 2

(c) Applying elementary row operations, transform the following matrix into reduced echelon form :

$$\begin{bmatrix} 0 & 3 & -6 & 6 & 4 & -5 \\ 3 & -7 & 8 & -5 & 8 & 9 \\ 3 & -9 & 12 & -9 & 16 & 15 \end{bmatrix}$$

(d) Solve the equation
$x^6+x^5+x^4+x^3+x^2+x+1=0$
with the help of De Moivre's theorem.

5. Answer ***any one*** of the following : 14×1=14

(a) (i) Let H be a subspace of a finite-dimensional vector space V. Prove that H is finite-dimensional and $\dim H \leq \dim V$. 8

(ii) Let $b_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, b_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, x = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$ and $B = \{b_1, b_2\}$ be a basis for $\mathbb{R}^2$. Find the coordinate vector $[x]_B$ of x relative to B. 6

(b) (i) For any integer $n \geq 1$, prove that $2^{2n} - 1$ is divisible by 3. 6

(ii) If p is a prime and p does not divide the integer c then show that $c^{p-1} \equiv 1 \pmod{p}$. 8

(c) (i) Find the dimensions of the null space and column space of the matrix 7

$$\begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$$

(ii) Let $B = \{b_1, b_2, b_3, \ldots, b_n\}$ be a basis for a vector space V. Prove that the coordinate mapping $x \to [x]_B$ is a one-one linear transformation from V to $\mathbb{R}^n$. 7