Total number of printed pages–8

63/1 (SEM–1) CC1/MATHC1016

2024

MATHEMATICS

Paper : MATHC 1016

(Calculus)

Full Marks : 60

Pass Marks : 24

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Choose the correct answer : 1×5=5

(a) The polar equation of a conic $\frac{l}{r} = 1 + e\cos\theta$ will represent an ellipse if –

(i) $e = 1$

(ii) $e = 0$

(iii) $e < 1$

(iv) $e > 1$

(b) If $\vec{a}, \vec{b}, \vec{c}$ be three vectors then $\vec{a} \times (\vec{b} \times \vec{c})$ equals to

(i) $(\vec{a}.\vec{c})\,\vec{b} - (\vec{a}.\vec{b})\,\vec{c}$

(ii) $(\vec{b} \times \vec{c}) \times \vec{a}$

(iii) $(b \times \vec{c}) . \vec{a}$

(iv) $(\vec{c}.\,\vec{a}).\,\vec{b} - (\vec{b}.\vec{c})\,\vec{a}$

(c) $\int_0^{na} f(x)dx = n\int_0^{a} f(x)\,dx$, if –

(i) $f(x) = f(a-x)$

(ii) $f(x) \neq f(a)$

(iii) $f(x) = f(a)$

(iv) $f(x) = f(a+x)$

(d) A curve $y = f(x)$ is concave upwards, if –

(i) $\frac{d^2y}{dx^2} > 0$

(ii) $\frac{d^2y}{dx^2} < 0$

(iii) $\frac{d^2y}{dx^2} \leq 0$

(iv) $\frac{d^2y}{dx^2} = 0$

(e) The value of $\cosh 2x$ is :

(i) $1 - \operatorname{sech}^2 x$

(ii) $1 + \operatorname{sech}^2 x$

(iii) $\cosh^2 x - \sinh^2 x$

(iv) $\cosh^2 x + \sinh^2 x$

(f) $D^n(x^n)$ is equal to –

(i) $\underline{|n-1}$

(ii) $n\,\underline{|n+1}$

(iii) nx^{n-1}

(iv) $\underline{|n}$

(g) If the equation
$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$
represents a hyperbola, then

(i) $\Delta \neq 0,\ h^2 - ab < 0$

(ii) $\Delta \neq 0,\ h^2 - ab > 0$

(iii) $\Delta \neq 0, \quad h^2 - ab = 0$

(iv) $\Delta = 0, \quad h^2 - ab \neq 0$

(h) The value of $\int_{-a}^{a} x^2 f(x) dx = 0$, if –

(i) $f(x)$ is even

(ii) $f(x)$ is odd

(iii) $f(x)$ periodic

(iv) $f(x)$ None of the above

(i) $\cosh x$ is equal to

(i) $\dfrac{e^x + e^{-x}}{2}$

(ii) $\dfrac{e^x - e^{-x}}{2}$

(iii) $\dfrac{e^{ix} - e^{-ix}}{2i}$

(iv) $\dfrac{e^{ix} + e^x}{2}$

(j) If the vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar, then–

(i) $[\vec{a}, \vec{b}, \vec{c}] = 0$

(ii) $\vec{a} \times \vec{b} \times \vec{c} = 0$

(iii) $\vec{a}.\vec{c} \times \vec{b} \neq 0$

(iv) $\vec{a}.\vec{b}.\vec{c} = 0$

2. Answer the following questions : ***(any five)*** 2×5=10

(a) Find y_n if $y = \log(a+x)$

(b) Show that $\dfrac{d}{dx}(\sinh u) = \cosh u \dfrac{du}{dx}$

(c) Prove that $\cosh^2 x - \sinh^2 x = 1$

(d) Transform the equation $x^2 - y^2 = a^2$ to axes inclined $\pi/4$ to the original axes.

(e) Solve $\int_0^{\pi/2} \cos^5 x \, dx$

(f) Let $\vec{r}(t) = t^2 \hat{e} + e^t \hat{j} - (2\cos \pi t)\hat{k}$, then find $\lim_{t \to 0} \vec{r}(t)$.

(g) Evaluate : $\int_0^1 (2t\hat{i} + 3t^2 \hat{j} + \hat{k})\, dt$

3. Answer ***any five*** of the following questions : 5×5=25

(a) Find the horizontal and vertical asymptotes of the curve $y = \dfrac{x^2 - x - 2}{x - 3}$

(b) Prove that $\left(a-2, \dfrac{-2}{e^2}\right)$ is a point of inflexion of the curve $y = (x-a)e^{x-a}$.

(c) Evaluate : ***(any one)***

(i) $\lim\limits_{x \to 0} \dfrac{e^x - e^{-x} - 2x}{x - \sin x}$

(ii) $\lim\limits_{x \to 0} \dfrac{x - \tan x}{x^3}$

(d) A manufacturer estimates that when x units of a commodity are produced each month the total cost (in Rs.) will be

$C(x) = \dfrac{1}{8}x^2 + 4x + 200$

and they can be sold at a price at $p(x) = 49 - x$ rupees per unit.

Determine the level of production at which average cost is minimized.

(e) Evaluate : $\int_0^{\pi/4} \sin^2\theta \cos^4\theta \, d\theta$

(f) If an incident ray from the point $(-1, 2)$ parallel to axis of parabola $y^2 = 4x$ strikes the parabola, then find the equation of the reflected ray.

(g) An object moves with position vector

$\vec{R}(t) = \cos t\hat{i} + \sin t\hat{j} + t\hat{k}$

Find the tangential and normal components of the acceleration.

(h) Sketch the graph of the parabola

$x^2 = 12y$

(i) Find the length of the arc of the parabola $y^2 = 4ax, a > 0$, measured from the vertex to one extremity of the latus rectum.

4. Answer the following questions : ***(any two)*** 10×2=20

(a) *(i)* State and prove Leibnitz theorem for the *n*th order derivative of the product of two functions. 1+5=6

(ii) If $y = \sin^{-1}x$, then prove that

$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0$ 4

(b) If $I_n = \int_0^{\pi/4} \tan^n x\, dx$, then show that

$$I_n + I_{n-2} = \frac{1}{n-1}$$

and hence deduce that $I_3 = \frac{1}{2} - \frac{1}{2}\log 2$

5+5=10

(c) (i) Find the area of the surface that is generated by revolving the portion of the curve $y = x^3$ between $x = 0$ and $x = 1$ about x-axis.

5

(ii) Find the volume of the solid generated by revolving the region by $y = x - x^2$ and $y = 0$ about the x-axis by using Disk method.

5

(d) (i) Establish the polar equation of the form $\frac{l}{r} = 1 + e\cos\theta$

of a conic referred to a focus as a pole. 5

(ii) If $y = \cos ax + \sin ax$, then prove that $\frac{d^n x}{dx^n} = a^n[1 + (-1)^n \sin 2ax]^{1/2}$

5
