63 (FY)SEM–3/MAJ/MATMAJ2024

2024

MATHEMATICS

Paper : MATMAJ2024

[Differential Equations (ODE)]

Full Marks : 50

Pass Marks : 20

Time : Two hours

The figures in the margin indicate full marks for the questions.

1. Choose the correct answer : 1×5=5

a. The order of the differential equation

$$\frac{d^4x}{dt^4}+5\frac{d^2x}{dt^2}+3x=\sin t$$ is

(i) 1

(ii) 2

(iii) 3

(iv) 4

Contd.

b. Which one of the following ordinary differential equation is linear ?

(i) $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+6y^2=0$

(ii) $\frac{d^2y}{dx^2}+5\frac{dy}{dx}+6y=0$

(iii) $\frac{d^2y}{dx^2}+5\left(\frac{dy}{dx}\right)^3+6y=0$

(iv) $\frac{d^2y}{dx^2}+5y\frac{dy}{dx}+6y=0$

c. If the Wronskian of the functions $y_1(x)$ and $y_2(x)$ is zero, i.e: $W(y_1,y_2)=0$, then the functions $y_1(x)$ and $y_2(x)$ are called

(i) linearly dependent

(ii) non-linearly dependent

(iii) linearly independent

(iv) non-linearly independent

d. The particular integral of $(D^2-3D+2)y=e^{5x}$ is

(i) xe^{5x}

(ii) $\frac{e^{5x}}{5}$

(iii) $\frac{e^{5x}}{12}$

(iv) $\frac{e^{5x}}{25}$

e. The differential equation

$$\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+x^3y=0$$

is a second-order

(i) linear homogeneous

(ii) non-linear homogeneous

(iii) linear non-homogeneous

(iv) non-linear non-homogeneous

2. Answer the following questions : ***(any five)*** 2×5=10

a. Show that the function f defined for all real x by $f(x)=2\sin x+3\cos x$ is an explicit solution of the differential equation

$$\frac{d^2y}{dx^2}+y=0.$$

b. Find the integrating factor of

$$(1+y^2)dx=(\tan^{-1}y-x)dy$$

c. Examine the following differential equation is exact or not :

$$(3x+2y)dx+(2x+y)dy=0$$

d. Draw input-output compartmental diagram for CO_2.

e. Find the general solution of the differential equation

$$y''-4y=0.$$

f. Find the Wronskian of the functions x, x^2, x^3. Are they linearly independent ?

g. Write the formulation of the equation of limited growth with harvesting.

3. Answer the following questions : ***(any five)*** 5×5=25

a. Show the relation $x^2+y^2-25=0$ is an implicit solution of the differential equation $x+y\frac{dy}{dx}=0$ on the interval I defined by $-5<x<5$.

b. Solve : $(3x^2+4xy)dx+(2x^2+2y)dy=0$

c. Solve : $(x-4)y^4dx+x^3(y^2-3)dy=0$

d. Solve the initial value problem (IVP) in equation $\frac{dN}{dt}=-kN$,

with initial condition $N(t_0)=n_0$.

e. Find a differential equation for the amount of salt in the tank at any time t. (Note that concentration can be difined as the mass of salt per unit volume of mixture).

f. Using undetermined co-efficients method to solve

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + y = x^2 - 2x + 2.$$

g. Solve by the method of variation of parameter the equation

$$\frac{d^2y}{dx^2} + a^2y = \sec ax$$

h. Solve :

$$\left(D^2 - 2D + 5\right)y = e^{2x}\sin x.$$

4. Answer the following questions : ***(any one)*** 10×1=10

(a) *(i)* Solve the following differential equation by finding an integrating factor 5

$$(5xy + 4y^2 + 1)dx + (x^2 + 2xy)dy = 0$$

(ii) Write down Bernoulli's equation and solve it. 1+4=5

(b) Write the differential equation of drug assimilation model of a single cold pill and solve it. 2+8=10
