2022

(Held in 2023)

PHYSICS

Paper : PHYHC5116

(Quantum Mechanics and Applications)

Full Marks : 60

Pass Marks : 24

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct option from the following : 1×5=5

(a) If $\hat{A}$ and $\hat{B}$ be two commutating operators, then

(i) $[\hat{A}, \hat{B}] = -i\hbar$

(ii) $[\hat{A}, \hat{B}] = 0$

(iii) $[\hat{A}, \hat{B}] = -1$

(iv) $[\hat{A}, \hat{B}] = +1$

(b) Stationary states are those for which the probability density ρ is

(i) time-dependent

(ii) time-independent

(iii) space-dependent

(iv) space-independent

(c) Quantum mechanical operator of momentum p is

(i) $\frac{\hbar}{i}\nabla$

(ii) $\frac{-\hbar^2}{2m}\nabla^2$

(iii) $\frac{\hbar}{i}\nabla^2$

(iv) $\frac{\nabla^2}{8\pi}$

(d) In Schrödinger picture, the state vector ψ is

(i) time-dependent

(ii) time-independent

(iii) constant

(iv) None of the above

(e) The energy of an linear harmonic oscillator in the nth quantum state is

(i) $\left(n+\frac{1}{2}\right)\hbar\omega$

(ii) $n\hbar\omega$

(iii) $\frac{1}{2}\hbar\omega$

(iv) $\frac{5}{2}\hbar\omega$

2. Answer the following questions : 2×5=10

(a) What do you mean by parity operator? Whether it commutes with position operator? 1+1=2

(b) You have 10 eV photon and 10 eV electron. Which one has shorter wavelength? Explain your result.

(c) State the conditions of normalisation and orthogonality of two wave functions.

(d) Show that $[x^n, \hat{p}_x] = -i\hbar n x^{n-1}$, n is a positive integer.

(e) Distinguish between a classical and a quantum harmonic oscillator.

3. Answer any *five* of the following questions : 5×5=25

(a) If $\psi(x) = A e^{m\omega x^2/\hbar}$, find the expectation values of momentum and position. 2½+2½=5

(b) Show that $\hat{L}_z$ is Hermitian. Also prove that eigenvalue of Hermitian operator is real. 2+3=5 *(c)* State and prove Ehrenfest's theorem in one dimension. 1+4=5

(d) Show that the orbital angular momentum of the electron in hydrogen atom is $\alpha = \hbar\sqrt{l(l+1)}$, where l is the orbital quantum number.

(e) Discuss with necessary theory the splitting of sodium lines when *(i)* a weak magnetic field and *(ii)* a strong magnetic field is applied.

(f) Explain gyromagnetic ratio and Bohr magneton.

(g) Calculate possible angles between $\vec{L}$ and $\vec{S}$ for a d electron in one electron atom.

4. Answer any *two* of the following questions : 10×2=20

(a) What are symmetric and antisymmetric wave functions? Show that the maximum number of electrons in a shell is $2n^2$, in the light of distribution of electrons among the shells by using Pauli's principle. Define Hund's rule. 2+2+4+2=10

(b) A particle moving in one dimension is confined to move within $x = 0$ and $x = a$. Suppose the particle is in a state represented by the wave function

$$\psi(x) = A\sin\frac{\pi x}{a}$$

(i) Find out the value of A that will normalise ψ.

(ii) What are the possible values of p those will be observed?

(iii) What are the average values of p and p^2?

(iv) Estimate the uncertainty product $\Delta x \, \Delta p$ for the given state. Is it consistent with uncertainty principle? 2+2+2+(2+2)=10

(e) If R and T be the reflection coefficient and transmission coefficient of a particle incident at the boundary of a potential barrier of height V_0, then show that $R + T = 1$, for $E < V_0$.

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