

**63/1 (SEM-3) CC5/PHYHC3056**

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**( Held in 2023 )**

**PHYSICS**

**Paper : PHYHC3056**

**( Mathematical Physics—II )**

Full Marks : 60

Pass Marks : 24

**Time : 3 hours**

*The figures in the margin indicate full marks  
for the questions*

**1. Choose the correct option from the following :**

**1×5=5**

(a)  $\int_0^{2\pi} \sin nx \cos nx dx =$

(i) 0

(ii)  $\pi$

(iii)  $-\pi$

(iv)  $\infty$

- (b) The regular singular point of the differential equation

$$x(x-1)y'' + (3x-1)y' + y = 0$$

is

- (i)  $x = 1$   
 (ii)  $x = -1$   
 (iii)  $x = 0$   
 (iv) None of the above

- (c)  $\beta(l, m) =$

(i)  $\int_0^1 x^{l-1} (1-x)^{m-1} dx$

(ii)  $\int_0^{\infty} x^{l-1} (1-x)^{m-1} dx$

(iii)  $\int_{-\infty}^{\infty} x^{m-1} (1-x)^{l-1} dx$

- (iv) None of the above

- (d)  $\Gamma(0) =$

(i) 0

(ii) 1

(iii) -1

(iv)  $\infty$

- (e) If  $\delta x_1 = 2$  mm and  $\delta x_2 = 3$  mm are the errors for  $x_1$  and  $x_2$  respectively, then the error in the sum of  $x_1$  and  $x_2$  is

(i) 5

(ii) 6

(iii) 3.60

- (iv) None of the above

2. Answer the following questions : 2×5=10

- (a) Express Fourier series in complex form.
- (b) Find the values of the Legendre polynomials  $P_0(x)$  and  $P_1(x)$  using generating function.
- (c) Write the Laplace equation in spherical polar coordinate. What is its general solution?
- (d) Show that  $\Gamma(n+1) = n\Gamma(n)$ .
- (e) Two resistances  $(5.1 \pm 0.03) \Omega$  and  $(2.6 \pm 0.02) \Omega$  are connected in series in a circuit. Find the total resistance.

3. Answer any five from the following questions : 5×5=25

(a) Find the Fourier series of the function  
 $f(x) = x^2, -\pi \leq x \leq \pi.$

(b) Show that the Bessel's function satisfy the recurrence relation

$$J_{m-1}(x) + J_{m+1}(x) = \frac{2m}{x} J_m(x)$$

(c) Show that,  $x=0$  is a regular singular point of the given equation and hence find the indicial equation

$$2x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + (1-x^2)y = 0$$

(d) Show that

$$\beta(l, m) = \frac{\Gamma(l)\Gamma(m)}{\Gamma(l+m)}$$

(e) Solve by the method of variable separation

$$\frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial t} + u$$

where  $u(x, 0) = 6e^{-3x}.$

(f) Fit a straight line to the following data by the method of least square :

$x$	:	-1	0	1	2	3
$y$	:	0	1	2	9	26

(g) (i) What is the error in density of a cube when mass is uncertain by  $\pm 2\%$  and length is uncertain by  $\pm 1\%$ ?

(ii) The heights of 5 persons (in cm) are—

150.5, 170.0, 160.0, 161.0, 170.5

Find the standard error. 2+3=5

4. Answer any two of the following : 10×2=20

(a) For the series

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

Find  $a_0, a_n$  and  $b_n.$

(b) Solve the one-dimensional diffusion equation

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{h^2} \frac{\partial \psi}{\partial t}$$

by variable separation method for

$$\psi = 0 \text{ when } x = 0$$

$$\psi = 0 \text{ when } x = l$$

and also,  $\psi = \sin \frac{\pi x}{l} \forall t = 0.$

( 6 ) :

(c) Find the power series solution of the linear oscillation system

$$\frac{d^2y}{dt^2} + \omega^2 y = 0$$

in powers of  $t$ .

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