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63/1 (SEM–5) DSE1/PHYHE5016

2023

PHYSICS

Paper : PHYHE5016

(Advanced Mathematical Physics–I)

Full Marks : 60

Pass Marks : 24

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Choose the correct option from the following : ***(any five)*** 1×5=5

   (a) Let A and B be any two matrices. Then

   (i) $AB = BA$

   (ii) $AB \neq BA$

   (iii) $AB < BA$

   (iv) $AB > BA$

Contd.

(b) Each set of $(n+1)$ or more vectors of a finite dimensional vector space $V(F)$ of dimension n is

(i) linearly dependent

(ii) a basis of $V(F)$

(iii) a subspace of $V(F)$

(iv) linearly independent

(c) If A^{μ} and B_{μ} are the components of a contravariant and covariant vectors, then the product $A^{\mu}B_{\mu}$ is

(i) a scalar

(ii) a contravariant tensor of rank 2

(iii) a covariant tensor of rank 2

(iv) a mixed tensor

(d) The trace of the matrix $A = \begin{bmatrix} 9 & -4 \\ 8 & -2 \end{bmatrix}$ is

(i) 4

(ii) 17

(iii) 7

(iv) −6

(e) If A is an orthogonal matrix, then

(i) $A^{-1}.A = I$

(ii) $A.A^{-1} = I$

(iii) $A'.A^{-1} = I$

(iv) $A.A' = I$

(f) Let $T: R^3 \to R^3$. Then T is linear, if

(i) $T(x, y, z) = (x^2, 2y, -z)$

(ii) $T(x, y, z) = (2x, xy, 3z)$

(iii) $T(x, y, z) = (x+z, y-3x, 2z)$

(iv) $T(x, y, z) = (x+5, y-2, 4z)$

(g) The rank of the tensor a^{il}_{jkl} is

(i) 0

(ii) 2

(iii) 3

(iv) 5

(h) In the term $a_{pq}x^q$

(i) p is dummy index and q is free index

(ii) q is dummy index and p is free index

(iii) Both p and q are dummy index

(iv) Both p and q are free index

(i) The value of $\delta_p^q \, \delta_r^p$ is

(i) δ_r^p

(ii) δ_r^q

(iii) 1

(iv) 0

(j) The eigenvalues of a Hermitian matrix are

(i) real

(ii) imaginary

(iii) complex

(iv) ± 1

2. Answer ***any five*** of the following questions : 2×5=10

(a) Let $X = (4, 5)$ in R^2. Find the coordinate vector for X with respect to the basis $A = \{(1, 1), (-1, 2)\}$.

(b) Show that any square matrix can be expressed as the sum of two matrices, one symmetric and the other skew-symmetric.

(c) Prove that a matrix A and its transpose A' have the same eigenvalues.

(d) What do you mean by symmetric and anti-symmetric tensors ?

(e) Show that $curl\,(grad\,\phi) = 0$ using tensor analysis.

(f) Write down the law of transformation of co-ordinate for the tensors— 1+1=2

(i) A_{ij}

(ii) B_r^{pq}

(g) What is a group ? In a group G, if $a * a = a$, $a \in G$, then prove that $a = e$ where e is the identity element of G.

3. Answer ***any five*** of the following questions : 5×5=25

(a) Verify the Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ and hence find A^{-1}. 3+2=5

(b) Find the matrix representation of linear transformation T on $V_3(R)$ defined as $T(a, b, c) = (2b + c, c - 4b, 3a)$ corresponding to the basis $B = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$.

(c) Determine whether $\begin{bmatrix} \frac{\sqrt{2}}{2} & -i\frac{\sqrt{2}}{2} & 0 \\ i\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is a unitary matrix or not.

(d) State and prove the contraction theorem of tensor. 2+3=5

(e) If $A = \begin{bmatrix} 2+i & 3 & -1+3i \\ -5 & i & 4-2i \end{bmatrix}$, show that AA^* is a Hermitian matrix where A^* is the conjugate transpose of A.

(f) Show that the Kronecker and alternating tensors are related as

$$\varepsilon_{iks}\,\varepsilon_{mps} = \delta_{im}\delta_{kp} - \delta_{ip}\delta_{km}.$$

(g) Solve the coupled linear differential equations using matrix method

$$\frac{dx}{dt} = 3x + y$$

$$\frac{dy}{dt} = x + 3y$$

where, $x(0) = 3;\ y(0) = 5$.

(h) Using tensors prove that

$$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}.(\vec{A}.\vec{C}) - \vec{C}.(\vec{A}.\vec{B})$$

(i) Prove that the sum and difference of two tensors of the same rank and same type is also a tensor of the same rank and same type.

4. Answer ***any two*** of the following questions : 10×2=20

(a) (i) What do you mean by vector subspace? Determine whether or not W is a subspace of R^3 where W consists of all vectors (a, b, c) in R^3 such that $a + b + c = 0$. 1+5=6

(ii) What are the conditions for a set of vectors $X_1, X_2, \ldots\ldots\ldots\ldots X_n$ are said to be linearly dependent? Examine whether the set of vectors $x_1 = (3, 2, 7)$, $x_2 = (2, 4, 1)$ and $x_3 = (1, -2, 6)$ are linearly dependent. If so, find a relation between them. 1+3=4

Contd.

(b) Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$ and diagonalize it. 7+3=10

(c) Derive an expression for the moment of inertia tensor. Prove that it is asymmetric tensor of order two. 7+3=10

(d) Define a metric tensor. Determine it in case of spherical polar coordinates. 2+8=10