2022

(Held in 2023)

PHYSICS

Paper : PHYHE5016

(Advanced Mathematical Physics—1)

Full Marks : 60

Pass Marks : 24

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer from the following : 1×5=5

(a) The function

$y_1 = f_1(x),\ y_2 = f_2(x),\ y_3 = f_3(x), \ldots\ldots, y_n = f_n(x)$

are said to be linearly dependent on some interval, such that

$$c_1 y_1 + c_2 y_2 + \cdots\cdots + c_n y_n = 0$$

if

(i) $c_1 = c_2 = \cdots\cdots = c_n = 0$

(ii) $c_1 = c_2 = \cdots\cdots = c_n \neq 0$

(iii) $c_1 \neq c_2 \neq \cdots\cdots \neq c_n$

(iv) None of the above

(b) Let A and B be two square matrices of order (3×3) and (3×3) respectively. Then the order of the inner product of A and B is

(i) 1×3

(ii) 3×3

(iii) 9×9

(iv) 27×27

(c) If A be a Hermitian matrix, then $i(A^3+2A-10I)$ is a

(i) Hermitian

(ii) skew-Hermitian

(iii) symmetric

(iv) skew-symmetric

(d) If δ be the Kronecker delta such that

$$\delta_i^k = \begin{cases} 0 \text{ if } i \neq k \\ 1 \text{ if } i = k \end{cases}$$

then the value of δ_i^k in four-dimensional geometry is

(i) 1

(ii) 2

(iii) 3

(iv) 4

(e) The transformation of A_{kl}^{ij} is

(i) $\bar{A}_{kl}^{ij} = \left(\dfrac{\partial \bar{x}^i}{\partial x^p}\dfrac{\partial \bar{x}^j}{\partial x^q}\dfrac{\partial x^r}{\partial \bar{x}^k}\dfrac{\partial x^s}{\partial \bar{x}^l}\right)A_{rs}^{pq}$

(ii) $\bar{A}_{kl}^{ij} = \left(\dfrac{\partial x^i}{\partial \bar{x}^p}\dfrac{\partial x^j}{\partial \bar{x}^q}\dfrac{\partial \bar{x}^r}{\partial x^k}\dfrac{\partial \bar{x}^s}{\partial x^l}\right)A_{rs}^{pq}$

(iii) $A_{kl}^{ij} = \left(\dfrac{\partial \bar{x}^i}{\partial x^p}\dfrac{\partial \bar{x}^j}{\partial x^q}\dfrac{\partial \bar{x}^r}{\partial x^k}\dfrac{\partial \bar{x}^s}{\partial x^l}\right)\bar{A}_{rs}^{pq}$

(iv) $\bar{A}_{kl}^{ij} = \left(\dfrac{\partial x^i}{\partial \bar{x}^p}\dfrac{\partial \bar{x}^j}{\partial x^q}\dfrac{\partial x^r}{\partial \bar{x}^k}\dfrac{\partial \bar{x}^s}{\partial x^l}\right)A_{rs}^{pq}$

2. Answer the following questions : $2\times5=10$

(a) Are the vectors $X_1 = (1, 3, 4, 2)$, $X_2 = (3, -5, 2, 2)$ and $X_3 = (2, -1, 3, 2)$ linearly dependent? If so, express one of these as a linear combination of the others.

(b) If A and B are Hermitian matrices, then prove that $(AB+BA)$ is also a Hermitian.

(c) Using Cayley-Hamilton theorem, find the inverse of A, where

$$A = \begin{bmatrix} 1 & 2 \\ 2 & -4 \end{bmatrix}$$

(d) Prove that

$$\frac{\partial g_{ij}}{\partial x^k} = [ik, j] + [jk, i]$$

(e) If A^i and B_j are the components of a contravariant and covariant tensors each of rank one, find the rank of a mixed tensor C^i_j, where $C^i_j = A^i B_j$.

3. Answer any *five* of the following questions : 5×5=25

(a) What are scalar matrix and unit matrix? Give examples.

(b) State and prove the quotient law of tensor.

(c) Reduce the following matrix to the diagonal form :

$$A = \begin{bmatrix} 5 & 5 & -5 \\ 0 & 4 & -1 \\ 2 & 8 & -3 \end{bmatrix}$$

(d) Write the transformation laws for the following tensors :

(i) A^i_{jk}

(ii) B^{mn}_{ijk}

(e) Define unitary matrix. Show that

$$A = \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$$

is an unitary matrix.

(f) Define symmetric and antisymmetric tensors. Prove that any second rank tensor can be written as a sum of symmetric and antisymmetric tensor.

(g) What is a group? Show that every subgroup of a cyclic group is also cyclic.

4. Answer any *two* of the following questions : 10×2=20

(a) Find the eigenvalues and eigenvectors of matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

and verify the orthogonality of its eigen-vectors.

(b) A covariant tensor has xy, $2y - x^2$, xz as Cartesian components. Find its spherical and cylindrical components.

(c) What are the importances of transformations of coordinates? What is Minkowski space? Find the transformation of contravariant and covariant components of a vector.

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