63/1 (SEM–4) PHY HC 4086 (CC 8)

2022

PHYSICS

Paper : CC–8

(Mathematical Physics—III)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer from the following : 1×5=5

(a) The modulus of the complex number

$$\frac{1-i}{1+i}$$

is

(i) 0

(ii) 1

(iii) –1

(iv) None of the above

(b) The value of $F_s[f(x)] =$

(i) $\sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \sin sx \, dx$

(ii) $\sqrt{\frac{2}{\pi}} \int_{-\infty}^\infty f(x) \sin sx \, dx$

(iii) $\sqrt{\frac{1}{2\pi}} \int_0^\infty f(x) \sin sx \, dx$

(iv) None of the above

(c) The value of $F[f(x)] =$

(i) $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^\infty f(x) e^{-isx} \, dx$

(ii) $\sqrt{\frac{2}{\pi}} \int_{-\infty}^\infty f(x) e^{isx} \, dx$

(iii) $\sqrt{\frac{2}{\pi}} \int_0^\infty f(x) e^{isx} \, dx$

(iv) None of the above

(d) The value of Laplace transform $L(1) =$

(i) 1

(ii) 0

(iii) ∞

(iv) $\frac{1}{s}$

(Continued)

(e) The value of $L[t^{-1/2}] =$

(i) $\sqrt{\pi / s}$

(ii) 0

(iii) $\sqrt{\pi}$

(iv) None of the above

2. Answer the following questions : 2×5=10

(a) For the complex number $z = 3 - 4i$, find z^4. Given $\tan^{-1} \frac{4}{3} = 53 \cdot 13°$.

(b) State De Moivre's theorem.

(c) Find the Fourier transformation of

$$f(x) = 0 \text{ for } x < \alpha$$
$$= 1 \text{ for } \alpha < x < \beta$$
$$= 0 \text{ for } x > \beta$$

(d) Find $L[\sin t \cos t]$.

(e) Show that $L[f'(t)] = sL[f(t)] - f(0)$.

3. Answer any *five* of the following questions : 5×5=25

(a) State and prove Cauchy integral theorem.

(b) Find the Taylor series expansion about the origin for $f(z) = \ln(1 + z)$.

(Turn Over)

(c) Show that

$$F_c[x^{n-1}] = \sqrt{\frac{2}{\pi}} \frac{\Gamma(n)}{s^n} \cos n\frac{\pi}{2}$$

and hence find

$$F_c\left[\frac{1}{x^{1/2}}\right]$$ 4+1=5

(d) Prove that

$$F[af_1(x) + bf_2(x)] = aF_1(s) + bF_2(s)$$

(e) Find the value of—

(i) $F_c[f'(x)]$;

(ii) $F_s[f'(x)]$. 2½+2½=5

(f) Find $L[1/t(1-\cos at)]$.

(g) Find Laplace transform of

$$f(t) = \begin{cases} \sin 2t, & 2\pi < t < 4\pi \\ 0, & \text{otherwise} \end{cases}$$

4. Answer any *two* of the following questions : 10×2=20

(a) Evaluate

$$\int_0^{2\pi} \frac{d\theta}{a + b\cos\theta}, \quad a > b > 0$$

applying calculus of residues and hence find

$$\int_0^{2\pi} \frac{d\theta}{2 + \cos\theta}$$ 8+2=10

(b) Solve by Fourier sine transforms

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

condition—

(i) $u = 0$ when $x = 0, t > 0$;

(ii) $u = \begin{cases} 1, & 0 < x < 1 \\ 0, & x \geq 1 \\ \text{when} & t = 0 \end{cases}$

(iii) $u(x, t)$ is bounded.

(c) A resistance R in series with inductance L is connected with e.m.f. $E(t)$. The equation is

$$L\frac{di}{dt} + Ri = E(t)$$

If the switch is connected at $t = 0$ and disconnected at $t = a$, find current i in terms of t.

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