2022

(Held in 2023)

MATHEMATICS

Paper : MATHC3076

(PDE and Systems of ODE)

Full Marks : 60

Pass Marks : 24

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer : 1×5=5

(a) The degree of the differential equation

$$x\frac{\partial^2 z}{\partial x^2}+\left(y\frac{\partial z}{\partial y}\right)^{\frac{1}{3}}+kz=0$$

is

(i) 2

(ii) 4

(iii) 3

(iv) 0

(b) The operator

$$\frac{\partial^2 u}{\partial x^2} - 4\frac{\partial^2 u}{\partial x \partial y} + 2\frac{\partial^2 u}{\partial y^2}$$

represents

(i) ellipse

(ii) hyperbola

(iii) parabola

(iv) None of the above

(c) Which of the following represents one-dimensional heat equation?

(i) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

(ii) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = k\frac{\partial^2 u}{\partial t^2}$

(iii) $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial u}{\partial t}$

(iv) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{c^2}\frac{\partial u}{\partial t}$

(d) If the system of two linear differential equations in two unknown functions

$$\frac{dx}{dt} = a_{11}(t)x + a_{12}(t)y + F_1(t)$$
$$\frac{dy}{dt} = a_{21}(t)x + a_{22}(t)y + F_2(t)$$

is to be non-homogeneous, then

(i) $F_1(t) > F_2(t)$

(ii) $F_1(t) = F_2(t) = c, \ c \neq 0$

(iii) $F_1(t) = F_2(t) = 0 \ \forall t$

(iv) $F_1(t) \neq 0, \ F_2(t) \neq 0, \ \forall t$

(e) Which of the following is linear partial differential equation?

(i) $p^2 + q^2 = 1$

(ii) $pq = p + q$

(iii) $x^2 p^2 + y^2 q^2 = z^2$

(iv) $x^2 p + y^2 q = z$

2. Answer the following : 2×5=10

(a) Form the partial differential equation by eliminating a, b from

$$z = ax + by + a^2 + b^2$$

(b) Find the complementary function of the partial differential equation

$$(D^3 - 3D^2 D' + 4D'^3)z = e^{x+2y}$$

(c) Define complete integral of a partial differential equation.

(d) If D is a differential operator with respect to t, then find the value of

$$(3D^2+6D-2)e^t$$

(e) Transform the single linear differential equation

$$\frac{d^2x}{dt^2}+2\frac{dx}{dt}-24x=0$$

into a system of first-order differential equation.

3. Answer any *five* of the following : 5×5=25

(a) Solve :

$$x^2p+y^2q=z^2$$

(b) Show that

$$x=2e^{2t},\ x=e^{7t}$$

and $y=-3e^{2t},\ y=e^{7t}$

are the solution of homogeneous linear system

$$\frac{dx}{dt}=5x+2y$$

$$\frac{dy}{dt}=3x+4y$$

(c) Apply Jacobi's method to find the complete integral of

$$P_1^3+P_2^2+P_3-1=0$$

(d) Find the characteristic equations and the characteristic curves of

$$\frac{\partial^2u}{\partial x^2}-2\frac{\partial^2u}{\partial x\partial y}-8\frac{\partial^2u}{\partial y^2}=0$$

(e) Derive one-dimensional heat equation in the form

$$\frac{\partial^2u}{\partial x^2}=\frac{1}{c^2}\frac{\partial u}{\partial t}$$

(f) For initial value problem (IVP)

$$\frac{dy}{dx}+2y=2-e^{-4x},\ y(0)=1$$

use Euler's method with a step size $h=0{\cdot}1$ to find approximate values of the solution at $x=0{\cdot}1, 0{\cdot}2, 0{\cdot}3, 0{\cdot}4$ and $0{\cdot}5$.

(g) Solve the system

$$\frac{dx}{dt}=x+2y,\ \frac{dy}{dt}=3x+2y$$

4. Answer any *two* of the following : 10×2=20

(a) Classify and reduce the equation

$$\frac{\partial^2 z}{\partial x^2}+4\frac{\partial^2 z}{\partial x \partial y}-5\frac{\partial^2 z}{\partial y^2}+6\frac{\partial z}{\partial x}+3\frac{\partial z}{\partial y}-9z=0$$

to canonical form.

(b) To find the solution of the wave equation

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$$

using the method of separation of variables.

(c) Using the Runge-Kutta fourth-order method, find approximate value of y when $x=0\cdot1$ and $0\cdot2$, if $\frac{dy}{dx}=x+y$ with initial condition $y(0)=1$.

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