

Total number of printed pages-11

63/1 (SEM-5) DSE2/MATHE5026

2023

MATHEMATICS

Paper : MATHE5026

(Probability and Statistics)

Full Marks : 80

Pass Marks : 32

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Choose the correct answer : **(any six)**
1×6=6

(a) What best defines a conditional probability ?

(i) It is dependent on another event having occurred

(ii) It has a bivariate distribution pattern

(iii) It is normally distributed

(iv) It has a random component

Contd.

- (b) Two random variables X and Y are distributed according to

$$f(x, y) = \begin{cases} (x + y), & 0 \leq x \leq 1, \\ & 0 \leq y \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

The probability $(X + Y \leq 1)$ is

- (i) 0.33
 - (ii) 0.44
 - (iii) 0.11
 - (iv) 0.22
- (c) How many independent variables are there in a bivariate distribution ?
- (i) 1
 - (ii) 2
 - (iii) 3
 - (iv) 4
- (d) The first moment about origin of binomial distribution is
- (i) nP
 - (ii) nPq
 - (iii) $n(1 - P)$
 - (iv) $n(1 - P)q$

- (e) A box contains six red balls and four green balls. Four balls are selected at random from the box. What is the probability that two of the selected balls will be red and two will be green ?

- (i) $\frac{1}{35}$
 - (ii) $\frac{1}{14}$
 - (iii) $\frac{1}{9}$
 - (iv) $\frac{3}{7}$
- (f) Which of the following is a continuous distribution ?
- (i) Binomial distribution
 - (ii) Poisson distribution
 - (iii) Geometric distribution
 - (iv) Exponential distribution

(g) If a random variable X satisfies the Poisson distribution with a mean value of 3, then the probability that $X \geq 2$ is

(i) $1 - 2e^{-3}$

(ii) $4e^{-3}$

(iii) $1 + 4e^{-3}$

(iv) $1 - 4e^{-3}$

(h) The binomial distribution have number of parameters

(i) one

(ii) two

(iii) three

(iv) four

(i) Suppose (X_n) is a Markov chain with 3 states and transition probability matrix

$$\begin{pmatrix} 1/3 & 1/3 & 1/3 \\ 1/2 & 1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Then which of the following statements is true ?

(i) $\{X_n\}$ is irreducible

(ii) $\{X_n\}$ is recurrent

(iii) $\{X_n\}$ does not admit a stationary probability distribution

(iv) $\{X_n\}$ has an absorbing state

(j) Let $f(x, y)$ be the joint probability density function of two independent random variables X and Y . Then

(i) $f(x, y) = 1, \forall (x, y)$

(ii) $f(x, y) = f_X(x) \cdot f_Y(y)$

(iii) $f(x, y) = f_X(x) / f_Y(y)$

(iv) $f(x, y) = f_Y(y) / f_X(x)$

2. Answer the following questions : **(any five)**
2×5=10

- (a) Define joint probability density function.
- (b) Show that probability of *any* impossible event is zero.
- (c) If X and Y are two random variables, then show that
$$\text{cov}(X, Y) = E(XY) - E(X)E(Y).$$
- (d) What are the *four* measures of central tendency ?
- (e) If the random variable X follows binomial distribution with parameters n and P , then show that $E(X) = nP$.
- (f) Define axiomatic probability.
- (g) If F is the distribution function of a random variable X and $a < b$, then show that $P(a < X \leq b) = F(b) - F(a)$.

3. Answer **any six** of the following questions :
5×6=30

- (a) The joint density function of X and Y is given by
$$f(x, y) = e^{-x} e^{-y}, 0 < x < \infty, 0 < y < \infty$$

Find the joint moment generating function of X and Y .

(b) If A and B are independent events, then show that \bar{A} and \bar{B} are also independent events.

(c) A bag contains 6 white and 9 black balls. Four balls are drawn at a time. Find the probability for the first draw to give 4 black balls if the balls drawn in the first draw are not replaced before the second draw.

(d) A random variable X has the following probability function values of X :

X :	1	2	3	4	5	6	7
$P(X)$:	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

Find k and also evaluate $P(X < 6)$,
 $P(X \geq 6)$ and $P(0 < X < 5)$.

- (e) Derive the characteristic function of binomial distribution.
- (f) A coin is tossed until a head appears. What is the expectation of the number of tosses required ?
- (g) Derive moment generating function of normal distribution.

(h) Let X and Y be two jointly continuous random variables with joint PDE

$$f(x,y) = \begin{cases} cx^2y, & 0 \leq y \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

(i) Find the constant c

(ii) Find $P\left(Y \leq \frac{x}{2}\right)$

(i) Explain Markov chain.

(j) The probability density function of a random variable X is given by

$$f(x) = \begin{cases} 2(1-x) & \text{for } 0 < x < 1 \\ 0 & \text{, elsewhere} \end{cases}$$

(i) Show that

$$E(X^r) = \frac{2}{(r+1)(r+2)}$$

(ii) Use this result to evaluate

$$E[(2X+1)^2]$$

4. Answer the following questions : **(any two)**
10×2=20

(a) (i) Show that expected value of X is equal to the expectation of the conditional expectation of X given

$$Y \text{ i.e., } E(X) = E[E(X/Y)] . \quad 5$$

(ii) Discuss the effect of change of origin and scale on moment generating functions. 5

(b) State and prove central limit theorem for independent and identically distributed random variables with finite variance.
2+8=10

(c) (i) Show that the mean and variance of the Poisson distribution are equal. 5

(ii) Write the probability density function of a random variable X which follows normal distribution with mean μ and variance σ^2 . What is a standard normal variate? Find its mean and variance. 5

(d) (i) Define mean, moments, central moments, variance and standard deviation of a random variable X .

5

(ii) State and prove Bayes' theorem. 5

5. Answer the following questions : **(any one)**

14

(a) Derive Chapman-Kolmogorov forward and backward equations for the birth and death process.

7+7=14

(b) (i) The mean and variance of a binomial distribution are 4 and $\frac{4}{3}$ respectively. Find $P(X \geq 1)$.

7

(ii) Ten coins are thrown simultaneously. Use binomial distribution to find the probability of getting at least seven heads. 7

(c) (i) A probability curve $y = f(x)$ has a range from 0 to ∞ . If $f(x) = e^{-x}$, find the mean and variance. 7

(ii) Let X be a continuous random variate with probability density function

$$\begin{aligned} f(x) &= ax, & 0 \leq x \leq 1 \\ &= a, & 1 \leq x \leq 2 \\ &= -ax + 3a, & 2 \leq x \leq 3 \\ &= 0, & \text{elsewhere} \end{aligned}$$

Compute $P(X \leq 1.5)$.

7