2022

(Held in 2023)

MATHEMATICS

Paper : MATSE3012

(Analytical Geometry)

(Theory)

Full Marks : 50
Pass Marks : 20

Time : 2 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer : 1×5=5

(a) The equation $xy = 1$ represents

(i) an ellipse

(ii) a parabola

(iii) a hyperbola

(iv) a circle

(b) Which of the following is not central conic?

(i) An ellipse

(ii) A parabola

(iii) A hyperbola

(iv) A circle

(c) The focus and directrix of a parabola $y^2 = 8x$ are

(i) $(2, 0)$, $x = 2$

(ii) $(2, 0)$, $x = -2$

(iii) $(0, 2)$, $x = 4$

(iv) $(0, 2)$, $x = -2$

(d) What does $x^2 + y^2 = a^2$ represent in space?

(i) A circle

(ii) A great circle

(iii) A cylinder

(iv) An ellipse

(e) The eccentricity of the ellipse

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

is

(i) $\frac{4}{9}$

(ii) $\frac{4}{25}$

(iii) $\frac{25}{9}$

(iv) $\frac{4}{5}$

2. Answer the following questions : 2×5=10

(a) What is a great circle? If the radius of a sphere is R, then what is the radius of the corresponding great circle?

(b) Sketch the curve $x = 2(y-3)^2 + 3$.

(c) If e_1 and e_2 be the eccentricities of a hyperbola and its conjugate, show that

$$\frac{1}{e_1^2} + \frac{1}{e_2^2} = 1$$

(d) Find the value of a of the parabola $y^2 = 4ax$ passes through the point $(3, 2)$. Write down the focus and the equation of directrix.

(e) Sketch the surface

$$\frac{x^2}{4}+\frac{y^2}{64}=1$$

3. Answer the following questions (any *five*) : 5×5=25

(a) Find the equation of the tangents to the hyperbola $4x^2-9y^2=144$, which are perpendicular to the straight line $6x+5y=21$. If the straight line $y=x+p$ touches the hyperbola $9x^2-25y^2=225$, find its point of contact.

(b) Establish the relation among a, b and c in respect of an ellipse where the symbols have their usual meanings.

(c) Find the equation of cones with vertex at the origin and the guiding curve is given by $f(x, y)=0$, $z=k$.

(d) Find the equation of the cone whose vertex is (α, β, δ) and the guiding curve is the conic $z=0$, $y^2=4ax$.

(e) Identify the type of the conic section of the equation

$$5x^2+10xy+5y^2+4x+2y+2=0$$

Also find the vertex.

(f) Prove that the length of the focal chord of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ making an angle θ to its major axis is

$$\frac{2ab^2}{a^2\sin^2\theta+b^2\cos^2\theta}$$

(g) Reduce the following equation of a conic to its standard form :

$$14x^2-4xy+11y^2-44x-58y+71=0$$

4. Answer any *one* of the following : 10

(a) Identify the vertices, co-vertices, foci, center, major axis and minor axis of the ellipse

$$\frac{(x+3)^2}{4}+\frac{(y-2)^2}{36}=1$$

and sketch the ellipse. Also, find the lengths of axes and latus rectum. 5+3+2=10

(b) Write the conditions for which the general second-degree equation

$$ax^2+2hxy+by^2+2gx+2fy+c=0$$

represents a parabola, an ellipse and a hyperbola.

Show that the equation

$$9x^2 - 16y^2 + 18x + 32y - 151 = 0$$

represents a hyperbola. Find the coordinates of the center, foci, length of axes, eccentricity and length of latus rectum. 5+5=10

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