2022

(Held in 2023)

ECONOMICS

Paper : ECOHC3066

(Mathematical Methods for Economics—II)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer/Choose the correct option from the following : 1×6=6

(a) What is an adjoint matrix?

(b) If A is a matrix of order $m \times n$ and B is a matrix such that AB' and $B'A$ are both defined, then order of matrix B is

(i) $m \times n$

(ii) $n \times m$

(iii) $n \times n$

(iv) $m \times m$

(c) What do you mean by Lagrange multiplier?

(d) If an estimated production function is $Q = 5L^{0.5}K^{0.5}$, then the function is homogeneous of degree

(i) 1

(ii) 0

(iii) ∞

(iv) None of the above

(e) The general solution of

$$(xy\frac{dy}{dx} - 1) = 0$$

is

(i) $xy = \log x + c$

(ii) $\frac{y^2}{2} = \log x + c$

(iii) $\frac{x^2}{2} = \log y + c$

(iv) None of the above

(f) If A is a singular matrix, then which one of the following is true?

(i) $|A| = 1$

(ii) $|A| = 0$

(iii) $|A| > 1$

(iv) $|A| > 0$

2. Answer the following questions : 2×5=10

(a) If

$$A = \begin{bmatrix} 1 & 2 \\ 4 & -5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 3 & 2 & 3 \\ 4 & 6 & 6 \end{bmatrix}$$

then find AB.

(b) If the total cost function is given by $C = 1000 + 6x + 0{\cdot}5x^2$, where x is output, then find the marginal cost function.

(c) If $A = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$, then show that $(A')' = A$.

(d) A utility function is given by $U = x^3 - 3xy^2 + y^3$, then find the marginal utilities of x and y.

(e) What is a determinant? Can it exist for a non-square matrix?

3. Answer any *six* of the following questions : 5×6=30

(a) If a demand and a supply function are given by

$$Q_d = a - bP \qquad (a, b > 0)$$
$$Q_s = -c + dP \qquad (c, d > 0)$$

then analyze the effect of change in the slope of demand function b on equilibrium quantity Q.

(b) Show that the production function $Z = \sqrt[3]{x^2 y}$ has a unit elasticity of substitution.

(c) Given $P_1 = 80 - 3Q_1$ and $P_2 = 104 - 4Q_2$ with joint cost function $C = 50 - 10Q + 2Q^2$, where $Q = Q_1 + Q_2$. Determine profit maximizing output level and maximum profit.

(d) What is a rank of a matrix? Find the rank of the following matrix :

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$

(e) If

$$A = \begin{bmatrix} 1 & 2 & -3 \\ 4 & -5 & 6 \end{bmatrix} \text{ and } B = \begin{bmatrix} 3 & 2 & 3 \\ 4 & 6 & 6 \end{bmatrix}$$

then prove that $4(A + B) = 4A + 4B$.

(f) Solve the following differential equation :

$$\frac{dy}{dx} + 5y = 9$$

Given $y(0) = 4$.

(g) If the production function is given by $Q = 4L^{3/4}K^{5/4}$, then find elasticity of substitution.

(h) If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, then show that $A^2 - 4A - 5I = 0$.

(i) A consumer's utility function is given by $U = x^2\sqrt{y}$. Find the slope of the indifference curve.

4. Answer any *two* of the following questions : 10×2=20

(a) Given the demand function $P = 90 - 0{\cdot}5Q$ and cost functions $C_1 = 0{\cdot}5Q_1^2$ and $C_2 = 0{\cdot}5Q_2$, where $Q = Q_1 + Q_2$ of two firms I and II. Determine equilibrium output, profit and price.

(b) The demand function under monopolistic competition with advertisement expenditure is given by $P = 100 - 2Q + 5\sqrt{A}$, where P is price, Q is output and A is advertisement expenditure. If his total cost function $C = Q^2 - 5Q + 10 + A$, then find the values of Q and A that will maximize profit of the firm.

(c) The production function of a particular commodity is $Q = L^{0.64}K^{0.36}$. Show that the isoquant is negatively sloped and convex to the origin.

5. Answer any *one* of the following questions : 14

(a) If

$$A = \begin{bmatrix} 0{\cdot}3 & 0{\cdot}2 & 0{\cdot}3 \\ 0{\cdot}1 & 0{\cdot}3 & 0{\cdot}4 \\ 0{\cdot}2 & 0{\cdot}3 & 0 \end{bmatrix} \text{ and } F = \begin{bmatrix} 500 \\ 700 \\ 600 \end{bmatrix}$$

then find the sectoral outputs (x_1, x_2 and x_3).

(b) A utility function of a consumer is given by $U = (x+4)(y+3)$, where x and y are two commodities consumed by the consumer. His budget line is given by $150 = 5x + 6y$. Find the values of x and y that maximize the consumer's utility.

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