2018

PHYSICS

(Major)

Paper : 4.1

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Mathematical Methods—IV)

(*Marks* : 40)

1. Answer any *four* of the following questions : 1×4=4

(a) What is singular point in a second-order linear differential equation?

(b) The function $\{1-2xh+h^2\}^{-1/2}$ is known as generating function of Legendre polynomial. Why?

(c) Mention an application of Hermite polynomial which is used in physics.

(d) Find the value of

$$\frac{2}{5}p_3(x)+\frac{3}{5}p_1(x)$$

(e) The word UNIVERSITY is arranged randomly. Find the probability that both I do not come together.

(f) What is Gaussian distribution?

2. Answer any *three* of the following questions : 2×3=6

(a) What is the value of $P_{2n+1}(x)$?

(b) What is the value of

$$\int_{-1}^{+1} x^n P_n(x)\,dx\ ?$$

(c) A card is drawn from a well shuffled pack of playing cards. Find the probability that it is either a king or a spade.

(d) Define total probability.

3. Answer any *two* of the following questions : 5×2=10

(a) From the value of Legendre polynomial, prove that

$$x^2 = \frac{1}{3}[2P_2(x) + P_0(x)]$$

(b) Find the indicial equation of the Hermite equation

$$\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2xy = 0$$

(c) Determine the probable error for the Gaussian distribution and express it as a multiple of σ.

(d) Define mean and standard deviation.

4. Answer any *two* of the following questions : 10×2=20

(a) *(i)* Use Frobenious method to find the series solution of the equation

$$\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 + 2)y = 0$$ 5

(ii) Find the degree and order of the following equation : 5

$$\frac{d^2y}{dx^2} + \frac{2}{x}\frac{dy}{dx} + \frac{81}{x^4} = 0$$

(b) (i) Write the generating function for Hermite polynomial $H_n(x)$ and prove that

$$H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n} e^{-x^2}$$ 5

(ii) Prove that

$$\int_{-\infty}^{+\infty} e^{-x^2} H_m(x)\, H_n(x)\, dx = 0$$

if $m \neq n$. 5

(c) (i) Legendre polynomials $P_n(x)$ are defined by the generating function

$$g(x,t) = (1 - 2xt + t^2)^{-\frac{1}{2}} = \sum_{n=0}^{\infty} P_n(x)\, t^n \quad |t| < 1$$

Hence prove that

$$(2n+1)\, x\, P_n(x) = (n+1)\, P_{n+1}(x) + n P_{n-1}(x)$$ 6

(ii) Prove the following recurrence relation : 4

$$2x H_n(x) = 2n H_{n-1}(x) + H_{n+1}(x)$$

(d) (i) What is the standard deviation of the following series? 5

Measurement :	0–10	10–20	20–30	30–40
Frequency :	1	3	4	2

(ii) Given $P(A) = \frac{1}{2}$, $P(A \cup B) = \frac{3}{5}$ and $P(B) = p$. Find the value of p if A and B are mutually exclusive and independent. 5