# 2018

# MATHEMATICS

## ( Major )

Paper : 1.2

## ( Calculus )

*Full Marks* : 80

*Time* : 3 hours

*The figures in the margin indicate full marks for the questions*

1. Answer the following questions : 1×10=10

(a) Write $n$th derivative of $\log(ax+b)$.

(b) If $z = f(y/x)$, what is the value of

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$$

(c) Write the formula for radius of curvature of a Cartesian curve.

(d) Two curves $y = f(x)$ and $y' = g(x)$ intersect at the point $(x_1, y_1)$. Find the condition that they cut orthogonally.

(e) Define double point of a curve.

(f) What is the volume of the solid generated due to the revolution of the circle $x^2 + y^2 = 4$ about $X$-axis?

(g) Write subnormal to the curve $y^2 = 4ax$ at any point $(x, y)$.

(h) Write the value of $\int_0^{\pi/2} \cos^7 x\,dx$.

(i) Write the value of

$$\int_0^{\pi/2} \frac{\sin^3 x}{\sin^3 x + \cos^3 x}\,dx$$

(j) Write the maximum number of asymptotes of algebraic curve of $n$th degree.

**2.** Solve the following questions : 2×5=10

*(a)* If $y = e^{ax} \sin bx$, show that

$$y_2 - 2ay_1 + (a^2 + b^2)y = 0$$

*(b)* If $x = r\cos\theta$, $y = r\sin\theta$, prove that

$$\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} = 0, \; x \neq 0, \; y \neq 0$$

*(c)* Find the area of the region bounded by the parabola $y^2 = 4x$ and its latus rectum.

*(d)* Find the value of $\int_0^{\pi} x\cos^4 x \, dx$.

*(e)* Find the equation of tangent to the curve $y = be^{-x/a}$ at the point, where it crosses the axis of $y$.

**3.** Answer the following questions : 5×2=10

*(a)* If $y = \sin^{-1} x$, find $(y_n)_0$ where $n$ is odd.

*(b)* Obtain a reduction formula for $\int \sec^n x \, dx$.

**4.** Answer either part *(a)* or part *(b)* : 10

*(a)* *(i)* If $u = \tan^{-1} \frac{x^3 + y^3}{x - y}$, find the value of

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$$

*(ii)* Find the points of inflexion of the curve $y(a^2 + x^2) = x^3$. 5+5=10

*(b)* *(i)* If $u = f(x, y)$, where $x = r\cos\theta$, $y = r\sin\theta$, show that

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2$$

*(ii)* If the normal to the curve $x^{2/3} + y^{2/3} = a^{2/3}$ makes an angle $\phi$ with *x*-axis, show that its equation is

$$y\cos\phi - x\sin\phi = a\cos 2\phi.$$

5+5=10

**5.** Answer the following questions : 5×2=10

(a) Evaluate :

$$\int_0^{\pi/2} \log \sin x \, dx$$

(b) Find the area of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

**6.** Answer part (a) or part (b) : 10

(a) (i) Obtaining $n$th derivative of $x^{2n}$, prove that

$$1 + \frac{n^2}{1^2} + \frac{n^2(n-1)^2}{1^2 \cdot 2^2} + \frac{n^2(n-1)^2(n-2)^2}{1^2 \cdot 2^2 \cdot 3^2} + \cdots = \frac{\lfloor 2n}{(\lfloor n)^2}$$

(ii) Show that the area enclosed by the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ is $\frac{3}{8}\pi a^2$. 5+5=10

(b) (i) Find the asymptotes of the curve

$$x^3 + y^3 - 3axy = 0$$

(ii) Trace the curve $r = a(1 + \cos\theta)$. 5+5=10

( Turn Over )

**7.** Answer any *two* questions : 5×2=10

(a) Evaluate :

$$\int_0^{\pi/2} \frac{dx}{5+3\cos x}$$

(b) If $I_n = \int (a^2 + x^2)^{n/2} dx$, show that

$$I_n = \frac{x(a^2+x^2)^{n/2}}{n+1} + \frac{na^2}{n+1} I_{n-2}$$

(c) Evaluate :

$$\int \frac{dx}{(x^2-2x+1)\sqrt{x^2-2x+3}}$$

**8.** Answer the following questions : 5×2=10

(a) Integrate :

$$\int \frac{e^x dx}{e^x - 3e^{-x} + 2}$$

*Or*

$$\int \frac{dx}{(1-x)\sqrt{1-x^2}}$$

*(b)* Find the length of the arc of the parabola $y^2 = 4ax$ cut off by the line $y = 2x$.

*Or*

Find the surface area of the solid generated by revolving the cardioid $r = a(1 + \cos\theta)$ about the initial line.

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