2018

MATHEMATICS

(Major)

Paper : 1.1

(Algebra and Trigonometry)

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following as directed : 1×10=10

   (a) What is the condition that union of two subgroups of a group is again a subgroup of the group?

   (b) What is the order of element

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 6 & 4 & 2 & 5 & 1 & 7 & 8 & 9 \end{pmatrix}$$

   of the permutation group P_9?

(c) Is every subgroup of an Abelian group is normal?

(d) If I_n be a unit matrix of order n, then what is the matrix adj I_n?

(e) What is the normal form of the matrix $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$?

(f) If the non-singular matrix A is symmetric, then

(i) A is Hermitian

(ii) A is skew-Hermitian

(iii) A^{-1} is symmetric

(iv) A^{-1} is skew-symmetric

(Choose the correct answer)

(g) What is the rank of a non-singular matrix of order 3×3?

(h) Express the complex number $-1+i$ in its polar form.

(i) What is the relation between circular and hyperbolic functions of sine?

(j) What is the value of $\log_e i$?

2. Answer the following questions : 2×5=10

(a) If a is a generator of a cyclic group G, then show that a^{-1} is also a generator of G.

(b) If A is a symmetric matrix, then prove that adj A is also symmetric.

(c) With an example, show that a matrix which is skew-symmetric is not skew-Hermitian.

(d) If A and B be two equivalent matrices, then show that $\text{rank}\, A = \text{rank}\, B$.

(e) If $x + \frac{1}{x} = 2\cos\theta$, then show that

$$x^n + \frac{1}{x^n} = 2\cos n\theta$$

3. Answer the following questions : 5×2=10

(a) If H is a subgroup of a group G and N is a normal subgroup of G, then show that $H \cap N$ is a normal subgroup of H.

(b) Prove that n, nth roots of unity forms a series in GP.

Or

Show that

$$1 - \frac{2}{\lfloor 3} + \frac{3}{\lfloor 5} - \frac{4}{\lfloor 7} + \cdots \infty = \frac{1}{\sqrt{2}} \sin\left(\frac{\pi}{4} + 1\right)$$

4. Answer any *two* questions : 5×2=10

(a) If α, β, γ are the roots of the equation $x^3 + qx + r = 0$, then form an equation whose roots be $(\alpha - \beta)^2$, $(\beta - \gamma)^2$, $(\gamma - \alpha)^2$.

(b) Solve the equation by Cardon's method

$$x^3 + 6x^2 + 9x + 4 = 0$$

(c) If $A, B, \cdots, L; a, b, \cdots, l; m \in R$, then prove that

$$\frac{A^2}{x-a} + \frac{B^2}{x-b} + \cdots + \frac{L^2}{x-l} = x + m$$

has all its roots real.

5. Answer either *(a)* or *(b)* : 10

(a) Prove that a mapping $f : X \to Y$ is one-one onto iff there exists a mapping $g : Y \to X$ such that $g \circ f$ and $f \circ g$ are identity maps on X and Y, respectively.

(b) Show that an equivalence relation R in a non-empty set S determines a partition of S and conversely, a partition of S defines an equivalence relation in S.

6. Answer either *(a)* or *(b)* :

(a) If H and K be two subgroups of a group G, then prove that HK is a subgroup of G iff $HK = KH$. $[HK = \{hk : h \in H, k \in K\}]$ 10

(b) Prove that order of each subgroup of a finite group is a divisor of the order of the group. Hence prove that if G is a finite group of order n and $a \in G$, then $a^n = e$. 6+4=10

7. Answer either *(a)* or *(b)* :

(a) If $\tan(\alpha + i\beta) = x + iy$, then find x and y. Hence show that $x^2 + y^2 + 2x\cot 2\alpha = 1$. 10

(b) *(i)* If $x < \sqrt{2} - 1$, then prove that

$$2\left(x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \cdots\right) = \frac{2x}{1-x^2} - \frac{1}{3}\left(\frac{2x}{1-x^2}\right)^3 + \frac{1}{5}\left(\frac{2x}{1-x^2}\right)^5 - \cdots$$

(ii) Show that

$$\frac{\pi}{12} = \left(1 - \frac{1}{3^{1/2}}\right) - \frac{1}{3}\left(1 - \frac{1}{3^{3/2}}\right) + \frac{1}{5}\left(1 - \frac{1}{3^{5/2}}\right) - \cdots \infty$$

5+5=10

8. Answer either *(a)* or *(b)* :

(a) If A and B are two square matrices of the same order, then prove that

$$\text{adj } (AB) = (\text{adj } B) \cdot (\text{adj } A)$$

Verify it for the matrices

$$A = \begin{bmatrix} 1 & -2 \\ 3 & -1 \end{bmatrix}, \; B = \begin{bmatrix} 6 & 1 \\ -1 & 3 \end{bmatrix}$$ 6+4=10

(b) What is normal form of matrix of a rank r? Find the rank of the matrix

$$A = \begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}$$

by reducing it to normal form. 2+8=10

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