2018

MATHEMATICS

(Major)

Paper : 6.2

(Numerical Analysis)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×7=7

(a) What do you mean by 'normalized floating point representation' of real numbers?

(b) Define 'round-off' error.

(c) Write down the approximate representation of $\frac{2}{3}$ correct to four significant figures and find the absolute error.

(d) Give the relationship between the operators E and Δ.

(e) Evaluate $E^2 x^2$ when $h = 2$.

(f) Show that $E\nabla \equiv \nabla E \equiv \Delta$.

(g) Write the general quadrature formula in numerical integration.

2. Answer the following questions : 2×4=8

(a) Explain briefly the importance of numerical differentiation method.

(b) Determine the number of significant figures in 8·1205 given its absolute error as $0 \cdot 3 \times 10^{-2}$.

(c) Evaluate :

$$\left(\frac{\Delta^2}{E}\right) x^3$$

(d) Write the numerical differentiation formulae for finding the first and second derivatives of a function $f(x)$ at a point x near the beginning of a given set of tabulated values.

3. Answer the following questions : 5×3=15

(a) Find the absolute, relative and percentage error when $\frac{3}{7}$ is approximated by 0·4286.

(b) Using the method of separation of symbols, prove that

$$u_x = u_{x-1} + \Delta u_{x-2} + \Delta^2 u_{x-3} + \cdots + \Delta^{n-1} u_{x-n} + \Delta^n u_{x-n}$$

where $u_{x+h} = E^h u_x$.

Or

In an examination, the number of candidates who obtained marks between certain limits were as follows :

Marks	*No. of candidates*
0–19	41
20–39	62
40–59	65
60–79	50
80–99	17

Find the number of candidates who obtained less than 70 marks using a suitable interpolation formula.

(c) Find the polynomial of the lowest possible degree which assumes the values 3, 12, 15, –21 when x takes the values 3, 2, 1, –1 respectively.

Or

Construct Lagrange's interpolating polynomial using the following data :

x :	40	45	50	55
$y = f(x)$:	15·22	13·99	12·62	11·13

4. Answer any *one* part :

(a) *(i)* Given $u_{20} = 24$, $u_{24} = 32$, $u_{28} = 35$, $u_{32} = 40$, find u_{25} by Bessel's formula.

(ii) Find $f'(4)$ and $f''(4)$ from the following data :

x :	1	2	4	8	10
$f(x)$:	0	1	5	21	27

5+5=10

(b) *(i)* Use Stirling's formula to find a polynomial of degree three or less which takes the following values of the function u_x :

x :	4	6	8	10
u_x :	1	3	8	20

(ii) Find the value of

$$\int_0^1 \frac{x^2}{1+x^3}\,dx$$

using Simpson's $\frac{1}{3}$rd rule, dividing the range into four equal parts. 5+5=10

(Continued)

5. Answer any *one* part :

(a) *(i)* Evaluate

$$\int_4^{5\cdot2} \log_e x \, dx$$

by Weddle's rule. Also compute the error.

(ii) Derive Simpson's $\frac{1}{3}$rd rule from Newton-Cotes quadrature formula.

5+5=10

(b) *(i)* A curve is drawn to pass through the points given by the following table :

x	:	1	1·5	2	2·5	3	3·5	4
y	:	2	2·4	2·7	2·8	3	2·6	2·1

Estimate the area bounded by the curve, the x-axis and the lines $x = 1$ and $x = 4$.

(ii) The following table gives the values of acceleration f of a particle in cm/sec^2 at equal interval of time t in sec.

t	:	0·0	0·5	1·0	1·5	2·0
f	:	0·3989	0·3521	0·2420	0·1295	0·0540

Find the velocity of the particle at $t = 2$ sec.

5+5=10

6. Answer any *one* part :

(a) *(i)* Derive the rate of convergence of the secant method.

(ii) Use Newton-Raphson method to find a positive root of the equation $e^x - 3x = 0$ correct to four decimal places. 5+5=10

(b) *(i)* Find an approximate real root of the equation $x^3 - x - 1 = 0$ using bisection method. Perform four iterations.

(ii) Using Regula-Falsi method, find the approximate real root of the equation

$$x \log_{10} x - 1 \cdot 2 = 0$$

correct to five decimal places. 5+5=10

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