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3 (Sem-1). MTH M1 (BU)

2018

MATHEMATICS

(Major)

Paper : 1.1

(Calculus)

Full Marks – 80

Time – Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions as directed :
1×10=10

(a) The value of $\sin(ix)$ is

(i) $-i\sinh x$ (ii) $i\sinh x$

(iii) $i\sin x$ (iv) zero

Choose the correct answer.

(b) Write down the n th derivative of $\sin ax$.

[Turn over

(c) A curve $y = f(x)$ is convex upwards if

$$\frac{d^2y}{dx^2} \text{ is}$$

(i) positive

(ii) negative

(iii) zero

(iv) None of these

Choose the correct answer.

(d) The curve $y = f(x)$ is symmetrical about the line $y = x$ if

(i) x and y are interchanged

(ii) x is replaced by $-y$

(iii) x is replaced by $-x$

(iv) x and y are replaced by $-x$ and $-y$ respectively.

Choose the correct answer.

(e) If $\vec{a}, \vec{b}, \vec{c}$ be three vectors then the product $\vec{a} \times (\vec{b} \times \vec{c})$ equals to

(i) $(\vec{a} \cdot \vec{c}) \vec{b} - (\vec{b} \cdot \vec{c}) \vec{a}$

(ii) $(\vec{a} \cdot \vec{c}) \vec{b} - (\vec{a} \cdot \vec{b}) \vec{c}$

(iii) $(\vec{a} \times \vec{b}) \times \vec{c}$

(iv) $(\vec{b} \times \vec{c}) \times \vec{a}$

Choose the correct answer.

(f) Write the formula for $\frac{d}{dt}(\phi \bar{a})$, $\phi = \text{constant}$.

(g) If the axes turned through the same angle θ so that the equation

$$ax^2 + 2hxy + by^2 = 0$$

changed into one which is free from the term involving xy then θ is equal to

(i) $\frac{1}{2} \tan^{-1} \frac{2h}{a-b}$

(ii) $\tan^{-1} \frac{2h}{a-b}$

(iii) $\tan^{-1} \frac{a-b}{2h}$

(iv) $(a-b) \tan^{-1} h$.

Choose the correct answer.

(h) The general second degree equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, where $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$. The equation represents a parabola if

(i) $h^2 = ab$, $\Delta = 0$

(ii) $h^2 < ab$, $\Delta \neq 0$

(iii) $h^2 > ab$, $\Delta \neq 0$

(iv) $\Delta = 0$

Choose the correct answer.

(i) The polar equation of a conic is

$$\frac{l}{r} = 1 + e \cos \theta. \text{ If } e < 1 \text{ then it represents}$$

- (i) a parabola (ii) an ellipse
(iii) a hyperbola (iv) a straight line

Choose the correct answer.

(j) If a particle moving along a curve and at some instant t , the particle at P whose intrinsic coordinates are (s, ψ) and the radius of curvature is ρ then the tangential acceleration at P will be

- (i) $\frac{dv}{dt}$ (ii) $\frac{v^2}{\rho}$
(iii) $\frac{ds}{d\psi}$ (iv) $\frac{ds}{dt}$

given that \bar{v} is the velocity vector at P .

Choose the correct answer.

2. Answer the following questions : $2 \times 5 = 10$

(a) Prove that : $\cosh^2 x - \sinh^2 x = 1$.

(b) Find the point of inflexion of the curve

$$x = \log \left(\frac{y}{x} \right).$$

- (c) Evaluate the following by using L'Hospital's rule :

$$\lim_{x \rightarrow 0} \frac{\tan x - x}{x - \sin x}$$

- (d) Find the reduction formula for

$$\int_0^{\pi/4} \tan^n x \, dx.$$

- (e) The value of the scalar triple product is zero if two of its vectors are parallel. Prove it.

3. Answer the following questions : $5 \times 2 = 10$

- (a) Evaluate $e^{x+1} \cos x$ by using Leibnitz's rule.
- (b) The lemniscate $r^2 = a^2 \cos 2\theta$ revolves about a tangent at the pole. Show that the surface of the solid generated is $4\pi a^2$.

Or

Find the volume by slicing using the disc method of the region bounded by $y = x^3$, $y = x$ about the x axis.

4. Answer any *two* of the following questions :

$$5 \times 2 = 10$$

- (a) Define asymptote. Find the asymptotes of the curve $x^3 + y^3 - 3axy = 0$.
- (b) Determine the length of an arc of the cycloid $x = a(\theta + \sin \theta)$, $y = a(1 - \cos \theta)$ measured from the vertex.
- (c) Show that the velocity of a planet at any point of its orbit is the same as it would have been fallen to the point from rest at a distance from the sun equal to the length of the major axis.

5. Answer either (a) or (b) :

$$10 \times 1 = 10$$

(a) (i) If the equation

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$
represents a central conic then find its centre.

(ii) Find the polar equation of a conic in the form $\frac{l}{r} = 1 - e \cos \theta$.

(b) (i) Prove that

$$\hat{i} \times (\bar{a} \times \hat{i}) + \hat{j} \times (\bar{a} \times \hat{j}) + \hat{k} \times (\bar{a} \times \hat{k}) = 2\bar{a}.$$

(ii) Suppose P be the position of a moving particle at time t and \bar{r} be its position vector with respect to a fixed point O as origin then find the tangential component of acceleration at P.

6. Answer any *one* of the following : $10 \times 1 = 10$

(a) Trace the curve (witch of Agnesi)

$$xy^2 = 4a^2 (2a - x).$$

(b) Trace the curve (Cardioid)

$$r = a (1 - \cos \theta).$$

7. Answer any *one* of the following : $10 \times 1 = 10$

(a) Describe the techniques of sketching a parabola.

(b) Prove that the time taken by the earth to travel over half of its orbit remote from the sun separated by the minor axis is two days more than half the year. The eccentricity of the orbit is $\frac{1}{60}$.

8. Answer any *one* of the following : $10 \times 1 = 10$

(a) Find

(i) the average cost function and

(ii) the marginal cost function for the following total cost function at $Q = 3$ and at $Q = 5$ where

$$TC = 35 + 5Q - 2Q^2 + 2Q^3.$$

(b) What is the cylindrical shell method? Use the shell method to compute the volume of the solid traced out by rotating the region bounded by the X axis, the curve $y = x^3$ and the line $x = 2$ about the y axis.