

2018

MATHEMATICS

(Major)

Paper : 5.4

(Rigid Dynamics)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×7=7

(a) Write down the moment of inertia of a circular disc of mass M and radius a about an axis through its centre and perpendicular to its plane.

(b) Define radius of gyration of the rigid body about a line.

- (c) State the principal axes of a rigid body at a point O of the body.
- (d) A rigid body rotates with angular velocity $\vec{\omega}$ about a fixed axis and I denotes the moment of inertia of the body about the axis. Write down the expression for the kinetic energy of the body.
- (e) What do you mean by holonomic system?
- (f) Define conservative system.
- (g) State the theorem of the principle of conservation of energy of a rigid body.

2. Answer the following questions : $2 \times 4 = 8$

- (a) A rigid body consists of 3 particles of masses 3 units, 5 units and 2 units located at the points $(-1, 0, 1)$, $(2, -1, 3)$ and $(-2, 2, 1)$ respectively. Find the moments of inertia about (i) the y -axis and (ii) the z -axis.

(b) A body with one point fixed rotates with angular velocity $(0, 0, 2)$. Find the magnitude of the velocity of a particle of mass m of the body located at the point $(3, -4, 1)$.

(c) Find the number of degrees of freedom for a rigid body which has one point fixed but can move in space about this point.

(d) A rigid body of mass 2 units rotates with angular velocity $\vec{\omega} = (1, 1, -1)$ and has the angular momentum $\vec{\Omega} = (2, 3, -1)$. Find the kinetic energy of the body.

3. Answer the following questions : $5 \times 3 = 15$

(a) Find the moment of inertia of a hollow sphere of radius a and mass M about a diameter.

Or

If the moments and products of inertia of a body about three perpendicular concurrent axes are known, find the moment of inertia of the body about the line

$$\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$$

- (b) State d'Alembert's principle and use it to obtain the equations of motion of any rigid body.

Or

Show that the motion of a body about its centre of inertia is the same as it would be if the centre of inertia were fixed and the same forces acted on the body.

- (c) The lengths AB and AD of the sides of a rectangle $ABCD$ are $2a$ and $2b$ respectively. Show that the inclination to AB of one of the principal axes at A is

$$\frac{1}{2} \tan^{-1} \frac{3ab}{2(a^2 - b^2)}$$

4. Define impressed forces and effective forces. A uniform rod of length $2a$ revolves with uniform angular velocity ω about a vertical axis through a smooth joint at one extremity of the rod so that it describes a cone of semivertical angle α . Show that

$$\omega^2 = \frac{3g}{4a \cos \alpha} \quad 2+8=10$$

Or

- (a) A plank of mass m and length $2a$ is initially at rest along a line of greatest slope of a smooth plane inclined at an angle α to the horizon and a man of mass M , starting from the upper end walks down the plank so that it does not move, show that he will reach the other end in time

$$\sqrt{\frac{4Ma}{(m+M)g \sin \alpha}} \quad 6$$

- (b) A rod revolving on a smooth horizontal plane about one end, which is fixed, breaks into two parts; what is the subsequent motion of the two parts? 4
5. (a) A pendulum is supported at O and P is the centre of oscillation. Show that if an additional weight is rigidly attached at P , the period of oscillation is unaltered. 5
- (b) Use Lagrange's equations to find the differential equation for a compound pendulum which oscillates in a vertical plane about a fixed horizontal axis. 5
6. Derive the equations of motion of a rigid body in two dimensions when the forces acting on the body are finite. 10

Or

Write down the equations of motion of a rigid body in two dimensions under impulsive forces. Two equal uniform rods, AB and AC , are freely jointed at A , and are placed on a smooth table so as to be at right angles. The rod AC is struck by a blow at C in a direction perpendicular to itself. Show that the resulting velocities of the middle points of AB and AC are in the ratio $2 : 7$. $3+7=10$

The figures in the margin indicate full marks

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I. Answer the following questions

(a) Write the condition for the stability of a random system of particles.

(b) If the particles of a system are subject to a central force, show that the total angular momentum is conserved.