

2018

CHEMISTRY

(Major)

Paper : 5.1

(Quantum Chemistry)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

Symbols used signify their usual meanings

1. Answer in brief : 1×7=7

(a) Find the eigenvalue for the operator $\frac{d^2}{dx^2}$ if the function is $\cos 4x$.

(b) An operator \hat{O} is defined as $\hat{O}\psi = \lambda\psi$, where λ is a constant. Show whether the operator is linear or not.

(c) Show whether the function $\psi = e^{-x}$ is well-behaved or not within the interval $0 \leq x \leq \infty$.

Or

One of the conditions for a function to be well-behaved is that the function must be single-valued. State why the function has to be single-valued.

- (d) Draw a diagram to show the orientations of the orbital angular momentum of magnitude $\sqrt{2}\hbar$ in presence of the applied magnetic field in the z -direction.
- (e) Find the term symbol for an electron in the d -orbital.
- (f) Write the value of the angular function for s -orbital.

Or

Define the shape of an orbital.

- (g) For the ground-state H-atom, write the wave functions for the spin-orbital.

2. Answer the following questions : 2×4=8

- (a) Find the operator for total energy of a particle with mass m having coordinate (x, y, z) .

(b) Normalize the function $\sin \frac{n\pi x}{a}$ within the interval $0 \leq x \leq a$. Here $n = 1, 2, 3, \dots$.

Or

Show that the functions $\sin \frac{\pi x}{a}$ and $\cos \frac{\pi x}{a}$ are orthogonal within the interval $0 \leq x \leq a$.

(c) Let ψ_1 and ψ_2 be the eigenfunctions of the linear operator \hat{O} , having the same eigenvalue λ . Show that the linear combination of ψ_1 and ψ_2 is also an eigenfunction of \hat{O} having the same eigenvalue.

(d) Consider the following sets of quantum numbers :

(i) $n = 2, l = 0, m_l = 0$

(ii) $n = 2, l = 1, m_l = 0$

(iii) $n = 2, l = 1, m_l = +1$

(iv) $n = 2, l = 1, m_l = -1$

State which of these sets yield imaginary wave functions. State how real functions are obtained from these imaginary functions.

Or

Taking $2p_z$ -orbital as example, write why the p -orbital is dumbbell in shape.

3. What do you mean by complete wave function? Using Pauli's anti-symmetry principle, prove that no two electrons of an atom can have all the four quantum numbers alike. 1+4=5

Or

What do you mean by spin-orbit interaction? Write in brief about the Russell-Saunders scheme of coupling of angular momenta. Find the term symbols for the first excited state of He-atom. 1+2+2=5

4. Answer any *two* questions : 5×2=10

(a) Write the time-independent Schrödinger equation for H_2^+ . State Born-Oppenheimer approximation. Discuss how this approximation can be applied to separate the Schrödinger equation for H_2^+ into two equations—one for the nuclei and the other for the electron. 1+1+3=5

(b) Applying Hückel molecular orbital method, calculate the π -bond energy of ethene. Also find the expressions for the π -molecular orbitals. 3+2=5

- (c) Write how the molecular orbitals of a homonuclear diatomic molecule can be classified as σ or π . Which of these two is doubly degenerate and why? What is the basis of classifying the MOs as g or u ? 2+2+1=5

5. Answer either (a) and (b) or (c), (d) and (e) :

- (a) A particle of mass m is moving within a box of lengths a , b and c along x -, y - and z -axes respectively. The potential energy within the box is considered to be zero; outside the box it is considered to be infinity. Solve the time-independent Schrödinger equation for the particle to get the values of the wave function and the energy. Use these results to explain degeneracy. 4+2=6

- (b) Calculate the zero-point vibrational energy of HCl if its force constant is 516 Nm^{-1} . 4

Or

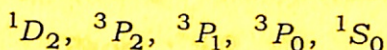
- (c) State the experimental observation of the photoelectric effect. Discuss how Einstein explained the observation. 3+2=5

- (d) A particle of mass m is moving in a one-dimensional box of length a , where potential energy is zero. Calculate the average kinetic energy of the particle. 3
- (e) An electron is confined to a molecule of length 10^{-9} m. Considering the electron to be a particle in one-dimensional box, where $V=0$, calculate its minimum energy. 2
6. Answer either (a), (b) and (c) or (d), (e) and (f) :
- (a) Define radial distribution function. Deduce an expression for the radial distribution function for non-s state. 1+3=4
- (b) Explain what you mean by space quantization. 3
- (c) Calculate the average value of potential energy of the electron of H-atom in the 1s state. 3

Or

- (d) What do you mean by radial function? Give the plots of radial function against r for $n=2$. State what information you can draw from these plots. 1+1+2=4

- (e) State Hund's rule of maximum multiplicity. For the $2p^2$ electrons of the ground-state C-atom, the following terms are obtained :



Using Hund's rule, state which of these terms will be the lowest in energy. $2+1=3$

- (f) Show that the maximum probability of finding the electron of the ground-state H-like atom is at $r = a_0/z$. 3

7. Answer either (a) and (b) or (c) and (d) :

- (a) Write the energy expressions for the bonding and the anti-bonding molecular orbitals of H_2^+ . Hence explain how the potential energy diagram is constructed. Write what information can be drawn from this diagram. $1+3+2=6$
- (b) Write the approximations of the Hückel molecular orbital theory. 4

Or

- (c) Write the ground-state molecular orbital wave function of H_2 . Hence explain the drawback of the molecular orbital theory in case of H_2 . State how Heitler and London modified the wave function. $1+3+1=5$

(d) Using LCAO-MO method, deduce the secular equations of H_2^+ . Hence deduce the expressions for the MO wave functions and their energies.

5

Standard integration :

$$\int_0^{\infty} x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$$
