2017

PHYSICS

(Major)

Paper : 2.1

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Mathematical Methods–II)

(*Marks* : 35)

1. Answer the following questions : 1×3=3

   (a) Define surface integral of a vector.

   (b) Write down the unit vectors $\hat{e}_\rho$, $\hat{e}_\phi$ and $\hat{e}_z$ of a cylindrical coordinate system in terms of $\hat{i}$, $\hat{j}$, $\hat{k}$.

   (c) Give graphical representation of Dirac delta function.

2. If $\vec{V} = (3x^2 + 6y)\hat{i} - 14yz\hat{j} + 20xz^2\hat{k}$, then evaluate $\int_C \vec{V} \cdot d\vec{r}$, where C is a straight line joining (0, 0, 0) and (1, 1, 1). 2

3. Answer any *two* of the following questions : 5×2=10

(a) (i) Show that $\Gamma(n+1) = n\Gamma(n)$. 2

(ii) Find the value of $\Gamma(\frac{1}{2})$. 3

(b) (i) Show that $\iint_S \vec{r} \cdot \hat{n}\, dS = 3$, over the surface S of the unit cube bounded by coordinate planes and the planes $x = 1$, $y = 1$, $z = 1$. 3

(ii) From the definition of Dirac delta function, show that

$$\int_{-\infty}^{+\infty} f(x)\, \delta(x-a)\, dx = f(a)$$ 2

(c) (i) Express differential operator $\vec{\nabla}$ in terms of orthogonal curvilinear coordinates. 3

(ii) If $\vec{r} = (u, v)$ represents a surface, then show that the square of the element of arc length on the surface is

$$dS^2 = E\,du^2 + 2F\,du\,dv + G\,dv^2$$

where $E = \frac{\partial \vec{r}}{\partial u} \cdot \frac{\partial \vec{r}}{\partial u}$, $F = \frac{\partial \vec{r}}{\partial u} \cdot \frac{\partial r}{\partial v}$ and

$G = \frac{\partial \vec{r}}{\partial v} \cdot \frac{\partial \vec{r}}{\partial v}$. 2

4. Answer any *two* of the following questions : 10×2=20

(a) *(i)* If V be the volume bounded by a closed surface S and $\vec{A}$ is a vector function of position with continuous derivatives, then prove that

$$\oiint_S \vec{A} \cdot \hat{n}\, dS = \iiint_V (\vec{\nabla} \cdot \vec{A})\, dV$$ 5

(ii) Prove the following identity : 3

$$\iiint_V (\phi \nabla^2 \psi - \psi \nabla^2 \phi)\, dV = \iint_S (\phi \vec{\nabla} \psi - \psi \vec{\nabla} \phi) \cdot d\vec{S}$$

(iii) If $\hat{n}$ be the unit outward drawn normal to any closed surface S, find the volume integral

$$\iiint_V \vec{\nabla} \cdot \hat{n}\, dV$$ 2

(b) (i) Express elements of area in orthogonal curvilinear coordinates. 3

(ii) Represent the vector

$$\vec{A} = 2y\hat{i} - z\hat{j} + 3x\hat{k}$$

in spherical polar coordinate. 5

(iii) Find the volume of a sphere of radius R. 2

(c) (i) Using vector integration, prove the equation of

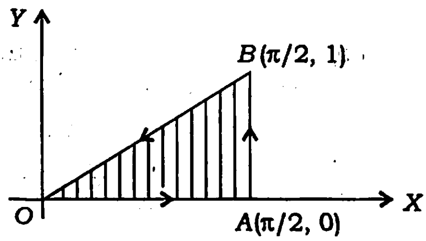
$$\vec{\nabla}\cdot(\rho\vec{v}) + \frac{\partial\rho}{\partial t} = 0$$

where ρ and $\vec{v}$ are the density and velocity of the fluid respectively. 5

(ii) State Green's theorem in the plane and using it evaluate

$$\oint_C (y - \sin x)\,dx + \cos x\,dy$$

where C is the triangle of the following figure : 5



GROUP—B

(Properties of Matter)

(*Marks* : 25)

5. Answer the following questions : 1×4=4

(a) Write down the relation connecting elastic constants Y, K and η.

(b) A rod is suspended vertically with its upper end rigidly fixed and is twisted at its lower end by means of a couple. For which layer of cylindrical rod shear is maximum?

(c) Why is Poiseuille's equation is not valid for blood flows through veins and capillaries?

(d) Define neutral surface of a beam.

6. Answer the following questions : 2×3=6

(a) Show that for a homogeneous and isotropic material the value of Poisson's ratio lies between −1 and ½, and is equal to ½ only when the material is incompressible.

(b) Write down the Poiseuille's assumptions to determine the rate of flow of liquid through a tube.

(c) How much work is required to break up a liquid drop of radius R into n equal small drops?

7. Answer any *one* of the following questions : 5

(a) (i) Show that simultaneous equal compression and extension at right angles to each other are equivalent to a shear. 3

(ii) Find the work done in joules in stretching a wire of cross-section 1 sq. mm and length 2 meters through 0·1 mm, if Young's modulus for the material of the wire is 2×10^{12} dynes/cm^2. 2

(b) (i) Find an expression for bending moment of a beam. 3

(ii) A steel rod of length 50 cm, width 2 cm and thickness 1 cm is bent into the form of an arc of radius of curvature 2 m. Calculate the bending moment. Young's modulus of the rod = 2×10^{11} N/m^2. 2

8. Answer *either* *(a)* and *(b)* *or* *(c)* and *(d)* : 10

Either

(a) Establish the relation between surface energy E and surface tension S of a liquid $E = S - T\frac{dS}{dT}$. 5

(b) *(i)* Show that angular oscillation of a torsional pendulum is simple harmonic and hence find the time period of the pendulum. 2

(ii) How can moment of inertia of a body be determined with the help of torsional pendulum? 3

Or

(c) Describe with necessary theory the rotating cylinder experiment for the measurement of coefficient of viscosity. 7

(d) An aluminium wire of length 2 m and radius 1 mm is twisted through 90°. Find the angle of shear at the surface, at the axis of the wire and at a point midway between the axis and surface. Calculate the torsional couple, if the modulus of rigidity is 5×10^{10} N/m^2. 3

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