

2017

PHYSICS

( Major )

Paper : 5.1

Full Marks : 60

Time : 3 hours

*The figures in the margin indicate full marks for the questions*

GROUP—A

( **Mathematical Methods** )

( Marks : 30 )

1. Answer the following questions : 1×4=4

(a) Give the Euler's formula.

(b) For the complex number  $z = 3 - 4i$ , find  $z^4$ , given that  $\tan^{-1} \frac{4}{3} = 53.13^\circ$ .

(c) Plot the number  $e^{(1 + \frac{\pi}{4}i)}$ .

(d) What is Argand plane or complex plane?

2. (a) State De Moivre's theorem. 2
- (b) Using De Moivre's theorem, show that  $e^{in\theta} = \cos n\theta + i \sin n\theta$ . 2
3. (a) Examine whether the function  $f(z) = (x + iy)^3$  is an analytic function or not. 3
- (b) Prove that  $\overline{z_1 \cdot z_2} = \overline{z_1} \cdot \overline{z_2}$ . 2

Or

Check the analyticity of the function  $f(z) = \ln z$  and hence find its derivative. 5

4. (a) State and prove Cauchy's integral theorem. 6
- (b) Using Cauchy's integral formula, evaluate  $\oint \frac{z-1}{z^2+1} dz$  around the contours—
- (i)  $|z-i|=1$
- (ii)  $|z|=2$  2+2=4

5. Answer either (a) and (b) or (c) and (d) :
- (a) State and prove Taylor's theorem. 5
- (b) Evaluate  $\oint_c \frac{dz}{z}$ , where  $c$  is a circle of unit radius. 2

(c) Obtain the residue of the function

$$f(z) = \frac{e^{iz}}{x^2 + a^2} \text{ at } z = ia. \quad 2$$

(d) Evaluate the given definite integral by

$$\text{the calculus of residues } \int_{-\infty}^{+\infty} \frac{dx}{(x^2 + a^2)^2}. \quad 5$$

GROUP—B

( Classical Mechanics )

( Marks : 30 )

6. Answer the following questions : 1×4=4

(a) What do you mean by constraints?

(b) What is a central force?

(c) What do you mean by a cyclic coordinate?

(d) What is areal velocity of a particle?

7. Answer any *three* of the following questions : .

2×3=6

(a) What are generalized coordinates?

(b) Define virtual displacement and discuss its significance.

- (c) Distinguish between Lagrangian and Hamiltonian formalisms.
- (d) Define reduced mass in the context of two-body central force problem.
- (e) Show that angular momentum is a constant of central force motion.

8. (a) State and establish d'Alembert's principle. 1+3=4

Or

Define Hamiltonian of a system. Under what conditions is it equivalent to energy? 1+3=4

(b) Show that a two-body central force problem can be reduced to one-body problem. 4

(c) Establish the Hamiltonian and hence the equation of motion of a simple pendulum. 5

Or

Establish the differential equation for the orbit under central force. 5

9. Find the equation of motion of a compound pendulum by using Lagrange's equation. Hence find an expression for period of oscillation for the pendulum. 5+2=7

Or

Derive Lagrange's equation of motion from Hamilton's principle for a conservative system. 7

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