

3 (Sem-3) PHY M 1

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PHYSICS

(Major)

Paper : 3.1

(Mathematical Methods—III and Electrostatics)

Full Marks : 60

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

GROUP—A

(Mathematical Methods)

(Marks : 25)

1. Answer the following questions : 1×3=3

(a) What do you mean by an idempotent matrix?

(b) Show that

$$A = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

is a self-adjoint matrix.

(c) Is $S = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ a symmetric matrix?

2. Show that

$$A = \begin{pmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{pmatrix}$$

is a nilpotent matrix.

2

3. Answer any *two* of the following questions :

5×2=10

(a) (i) Show that every square matrix can be expressed uniquely as the sum of a Hermitian and a skew-Hermitian matrix.

2

(ii) For the matrix $A = \begin{pmatrix} 1 & 2 \\ 3 & -5 \end{pmatrix}$, verify

the theorem

$$A (\text{adj } A) = (\text{adj } A) A = |A| I$$

3

(b) (i) Show that the trace of a product of two matrices is independent of the order of multiplication.

2

(ii) Determine the eigenvalues of the

$$\text{matrix } A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

3

(c) (i) Prove that the product of a singular matrix with its adjoint is the null matrix. 2

(ii) What is the rank of the matrix

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ -3 & -6 & -9 \end{pmatrix} ?$$
3

4. Answer either (a) and (b) or (c) and (d) : 5×2=10

(a) (i) For three Pauli spin matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \text{and}$$

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \text{show that}$$

$$\sigma_1 \sigma_2 = i \sigma_3. \quad 2$$

(ii) Prove that the modulus of each eigenvalue of a unitary matrix is unitary. 3

(b) Diagonalize the matrix

$$A = \begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$$
5

- (c) State Cayley-Hamilton theorem. Obtain the eigenvalue equation of the matrix

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{pmatrix}$$

and verify the Cayley-Hamilton theorem.

5

- (d) Using matrix, find the transformation equation for a 90 degree rotation about the z-axis of Cartesian coordinate system.

5

GROUP—B

(**Electrostatics**)

(Marks : 35)

5. Choose the correct answer/Answer the following questions : 1×4=4

- (a) Potential at a large distance from a quadrupole is proportional to

(i) $\frac{1}{r}$

(ii) $\frac{1}{r^2}$

(iii) $\frac{1}{r^3}$

(iv) $\frac{1}{r^4}$

- (b) How much electric energy is stored by a solid sphere of radius R and total charge Q?

(c) Electric field vector is

(i) rotational

(ii) irrotational

(d) Write Gauss' law of electrostatics in differential form.

6. Answer the following questions : 2×3=6

(a) Draw a schematic diagram of an electric octopole.

(b) Two charges 5 microcoulomb each are at the points (0, 3, 0) and (3, 0, 0) respectively. Find the electric intensity at (0, 0, 5).

(c) The electric field due to a short dipole at a point distant 1 cm from it on its perpendicular bisector is 1.5×10^{-11} volt/m. Find the dipole moment.

7. Using integral form of Gauss' law in electrostatics, find the electric field and potential at a distance z above the centre of a circular loop of radius R that carries a uniform line charge λ .

5

Or

Find an expression for the torque experienced by an electric dipole in external electric field. Hence show that the work done

in rotating the dipole from an initial position θ_1 to θ_2 is $W = -pE (\cos \theta_2 - \cos \theta_1)$. 3+2=5

8. Answer any two questions : 10×2=20

(a) (i) Using Gauss' law, find the electric field outside a uniformly charged solid sphere of radius R and total charge q . 3

(ii) Compute the curl and divergence of electric field given by

$$\vec{E}(x, y, z) = (x^2 + z^2 + 5)\hat{i} + (x^2 + y^2 - 9z)\hat{j} + (y^2 + z^2)\hat{k}$$
 2

(iii) Establish the Clausius-Mosotti equation for a linear dielectric material. 5

(b) (i) Define electric susceptibility, permeability and dielectric constant of material. 3

(ii) Using Laplace's equation, obtain an expression for potential inside a spherical capacitor of two concentric spheres having inner radius a and outer radius b . 3

(iii) State and prove uniqueness theorem. 4

- (c) (i) An isotropic dielectric is placed in an uniform electrostatic field E . Show that the field inside a spherical cavity in this direction is

$$E_i = E + \frac{P}{3\epsilon_0}$$

where P is the polarization. 5

- (ii) Write Poisson's equation. Solve Laplace's equation to find the potential at a distance r from the axis of an infinitely long conducting cylinder of radius R charged with a surface density σ . 1+4=5

- (d) (i) A point charge is situated near an infinite plane earthed conductor. Apply the method of electrical image to calculate—
1. the surface charge density induced on the plane;
 2. the force between the plane and the charge. 4+3=7

- (ii) An electron is at a distance 10 \AA from an infinite plane conductor. Calculate the force experienced by the electron and the work done in moving it to infinite distance away from the conductor. 3
