

2017

MATHEMATICS

( Major )

Paper : 1.1

( Algebra and Trigonometry )

Full Marks : 80

Time : 3 hours

*The figures in the margin indicate full marks for the questions*

1. Answer the following questions : 1×10=10

- (a) What is the order of  $A_n$ , alternating group of degree  $n$ ?
- (b) Is generator of a cyclic group always unique?
- (c) Does the set of all odd integers form a group with respect to addition?
- (d) Define Hermitian matrix.
- (e) What is normal form of a matrix?

- (f) What is the rank of a matrix, where every element of the matrix is unity?
- (g) If in a square matrix  $A$ ,  $|A|=0$ , then what is the value of  $|\text{adj } A|$ ?
- (h) Find the amplitude of the complex number  $-1-i$ .
- (i) What is the period of  $\sinh x$ ?
- (j) State Gregory series.

2. Give the answer of the following questions :

2×5=10

- (a) Can a non-Abelian group have an Abelian subgroup? Justify your answer.
- (b) Express the following matrix as a sum of symmetric and skew-symmetric matrix :

$$A = \begin{bmatrix} 3 & 4 \\ 2 & 1 \end{bmatrix}$$

- (c) Let  $A$  and  $B$  be two square matrices of order  $n$ . If  $AB=1$ , then prove that  $BA=1$ .
- (d) If the matrices  $A$  and  $B$  commute, then show that  $A^{-1}$  and  $B^{-1}$  also commute.

(e) If

$$x_r = \cos \frac{\pi}{2^r} + i \sin \frac{\pi}{2^r}$$

then prove that  $x_1 x_2 x_3 \dots \infty = \cos \pi$ .

3. Answer the following questions : 5×2=10

(a) Prove that every group of prime order is cyclic.

(b) Prove that  $i^i$  is completely real. Find its principal value.

Or

Prove that

$$\frac{1}{6} \sin^3 x = \frac{x^3}{\lfloor 3} - \frac{1}{\lfloor 5} (3^2 + 1) x^5 + \frac{1}{\lfloor 7} (3^4 + 3^2 + 1) x^7 + \dots$$

4. Answer any two questions : 5×2=10

(a) If  $\alpha, \beta, \gamma$  are the roots of the equation

$$x^3 - px^2 + qx - r = 0$$

then find the value of

$$\sum \left( \frac{\alpha}{\beta} + \frac{\beta}{\alpha} \right)$$

in terms of  $p, q$  and  $r$ .

- (b) Find the condition that the cubic

$$x^3 - px^2 + qx - r = 0$$

should have its roots in harmonic progression.

- (c) Using Descartes' rule of sign, show that when  $n$  is even, the equation  $x^n - 1 = 0$  has two real roots 1 and  $-1$  and no other real root, and when  $n$  is odd, the only real root is 1.

5. Answer any one question :

10

- (a) Let  $A$  be a non-empty set and let  $R$  be an equivalence relation in  $A$ . Let  $a$  and  $b$  be arbitrary elements in  $A$ . Then prove that—

(i)  $[a] = [b]$ , iff  $(a, b) \in R$ ;

(ii) either  $[a] = [b]$  or  $[a] \cap [b] = \phi$ .

- (b) Prove that an equivalence relation  $R$  in a non-empty set  $S$  determines a partition of  $S$  and conversely, a partition of  $S$  defines an equivalence relation in  $S$ .

6. Answer any one question : 10

(a) If  $H$  is a subgroup of  $G$ , then prove that there is a one-to-one correspondence between the set of left cosets of  $H$  in  $G$  and the set of right coset of  $H$  in  $G$ .

(b) Prove that a subgroup  $H$  of a group  $G$  is a normal subgroup of  $G$  if and only if the product of two right cosets of  $H$  in  $G$  is again a right coset of  $H$  in  $G$ .

7. Answer any one question : 10

(a) Find real and imaginary parts of

$$\sin^{-1}(\cos\theta + i\sin\theta) \quad (\theta \in \mathbb{R})$$

(b) If  $\tan(\theta + i\phi) = \cos\alpha + i\sin\alpha$ , prove that

$$\theta = \frac{n\pi}{2} + \frac{\pi}{4} \quad \text{and} \quad \phi = \frac{1}{2} \log_e \tan\left(\frac{\pi}{4} + \frac{\alpha}{2}\right)$$

8. Answer any one question : 10

(a) If  $A$  be any  $n$ -square matrix, then show that

$$A(\text{Adj}A) = (\text{Adj}A)A = |A|I_n$$

where  $I_n$  is the  $n$ -rowed unit matrix. Verify it for the matrix

$$A = \begin{bmatrix} 2 & -1 \\ -3 & -2 \end{bmatrix}$$

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(b) For what values of  $\eta$ , the equations

$$x + y + z = 1$$

$$x + 2y + 4z = \eta$$

$$x + 4y + 10z = \eta^2$$

have a solution? Solve them completely in each case.

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