

2017

MATHEMATICS

(Major)

Paper : 6.5

(Graph and Combinatorics)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×7=7

(a) If A and B are two sets and A' denotes the complement of A , then $A \cap (A \cup B)'$ is equal to

(i) A

(ii) B

(iii) ϕ

(iv) $A \cap B$

(b) In a class, there are 8 boys and 6 girls. The teacher wants to select either a boy or a girl to represent the class in a function. In how many ways the teacher can perform this job?

- (c) How many edges and vertices are there in the graph of $K_{2,5}$?
- (d) Draw the graph of K_5 .
- (e) Under what condition of n , the complete graph K_n is Eulerian?
- (f) If $r(m, n)$ is a Ramsey number, find $r(2, 2)$.
- (g) Let G be a graph with n edges and m vertices, then find

$$\deg(v_1) + \deg(v_2) + \dots + \deg(v_m)$$

2. Answer the following questions : 2×4=8

- (a) In a connected graph, show that number of odd degree vertices is even.
- (b) Let H be a subgraph of a graph G and \bar{H} be the complement of H in G . Draw the graphs H and \bar{H} from the graph G .
- (c) State the hand-shaking theorem. What is meant by source and sink in graph theory?
- (d) State only two properties of tree.

3. Answer any *three* of the following questions :

5×3=15

(a) For any natural number n and $0 < r \leq n$, prove that

$$P(n, r) = \frac{n!}{(n-r)!} \quad 5$$

(b) Suppose $S = \{1, 2, 3, \dots, n\}$, where $n \geq 1$. Find the number of ordered pairs (A, B) of subsets of S such that

(i) $A \neq B$

(ii) $A \cup B = S \quad 2+3=5$

(c) If $G = (V, E)$ is a (p, q) graph, then show that

$$\delta(G) \leq \frac{2q}{p} \leq \Delta(G)$$

where $\delta(G) = \min\{\deg(v_i) : v_i \in V\}$ and $\Delta(G) = \max\{\deg(v_i) : v_i \in V\}$. 5

(d) What do you mean by saying a connected graph? If G is not a connected graph, then show that \bar{G} is connected. 1+4=5

(e) Prove that the edge connectivity of a connected graph cannot exceed the minimum degree of G . 5

4. (a) Enumerate the number of non-negative integral solutions to the inequality

$$x_1 + x_2 + x_3 + x_4 + x_5 \leq 19 \quad 4$$

- (b) (i) What is combinatorial identity?
Also find the number of 3 combinations of

$$\{\infty \cdot a_1, \infty \cdot a_2, \infty \cdot a_3, \infty \cdot a_4\}$$

- (ii) Find the number of binary numbers with five 1s and three 0s.

- (iii) In how many ways can a king or a queen be drawn from an ordinary deck of playing cards? $2+2+2=6$

5. (a) If G is a simple graph with n vertices and k components, then show that G can have at most $\frac{(n-k)(n-k+1)}{2}$ edges. 6

- (b) Prove that every non-trivial tree has at least 2 vertices of degree 1. 4

Or

Prove that a vertex v in a connected graph G is a cut vertex if and only if there exist vertices u and w distinct from v such that every path connecting u and w contains the vertex v .

6. (a) Define Hamiltonian path and Hamiltonian circuit with an example for each. 4
- (b) Give an example of a graph that contains—
- (i) an Eulerian circuit that is also a Hamiltonian circuit;
 - (ii) neither an Eulerian circuit nor a Hamiltonian circuit. $1+1=2$
- (c) Let G be a connected graph which contains an Euler trail but not Euler circuit, then show that G has exactly two vertices of odd degree. 4

Or

When is a graph called Hamiltonian graph? Also show that every Hamiltonian graph is 2-connected.
