2017

MATHEMATICS

(Major)

Paper : 6.4

(Discrete Mathematics)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following as directed : 1×7=7

(a) Write the fundamental theorem of arithmetic.

(b) If the positive integers a and b are relatively prime, then

(i) $[a, b] = 1$

(ii) $a|b$

(iii) $(a, b) = 1$

(iv) None of the above

(Choose the correct option)

(c) If a and b are two positive integers such that $(a, b) = 1$, then

(i) $[a, b] = 1$

(ii) $[a, b] = ab$

(iii) $(a, b)\,[a, b] = 1$

(iv) None of the above

(Choose the correct option)

(d) State Wilson's theorem on congruence.

(e) State Fermat's last theorem.

(f) If $d = (a, b)$, $d|c$ and $ax + by = c$ has a particular solution x_0 and y_0, then write the other solution of the equation.

(g) Express 113 as a sum of two squares.

2. Answer the following questions : 2×4=8

(a) If $a|bc$ and $(a, b) = 1$, then show that $a|c$.

(b) What is the remainder when 5^{48} is divided by 12?

(c) Solve the linear congruence equation

$$6x \equiv 15 \pmod{21}$$

(d) Prove that the equation $x^4 + y^4 = z^4$ has no positive solution.

3. Answer the following questions : 5×3=15

(a) Prove that there is infinite number of primes of the form $4k+3$.

Or

Find the greatest common divisor g of the numbers 1819 and 3587 and find the integers x and y to satisfy $1819x+3587y=g$.

(b) Find the solution of the system of congruences

$$x \equiv 2 \pmod 3$$
$$x \equiv 3 \pmod 5$$
$$x \equiv 2 \pmod 7$$

(c) Prove that the equation $x^2+y^2=z^2$ has a primitive solution if and only if there exists $s, t \in N$, $s>t$, $(s, t)=1$ and one even and the other odd such that

$$x = x^2 - t^2$$
$$y = 2st$$
$$z = s^2 + t^2$$

Or

Examine whether the following Diophantine equation has solution. If it is solvable, find the solution :

$$172x+20y=1000$$

4. Answer *either (a) or (b)* : 10

(a) For each positive integer $n \geq 1$, prove that

$$\phi(n) = \sum_{d \mid n} \mu(d) \frac{n}{d} = n \prod_{p} \left(1 - \frac{1}{p}\right)$$

The symbols have their usual meanings.

(b) If $n = p_1^{s_1} p_2^{s_2} \cdots p_r^{s_r}$, p_i distinct primes and integers $s_i \geq 1$, then for each $r \geq 1$, prove that

$$\tau(n) = (s_1 + 1)(s_2 + 1) \cdots (s_r + 1)$$

5. Answer *either (a) or (b)* : 10

(a) Prove that every Boolean function which does not contain any constant is equivalent to function in disjunctive normal form (DNF). 10

(b) *(i)* Find the conjunctive normal form (CNF) for the function

$$f = x_1 x_2 x_3 + x_1 x_2' x_3 + x_1' x_2' x_3 + x_1' x_2' x_3'$$

(ii) Design a circuit connecting two switches and one bulb in such a way that either switch may be used to control the light independent of the order. 5+5=10

6. Answer *either (a) or (b)* : 10

(a) $\mathscr{A}$ is a statement form which contains another statement form $\mathscr{A}_1$. Further suppose $\mathscr{B}$ is obtained from $\mathscr{A}$ by substituting $\mathscr{B}_1$ for one or more occurrences $\mathscr{A}_1$, then show that $(\mathscr{A}_1 \leftrightarrow \mathscr{B}_1) \rightarrow (\mathscr{A} \leftrightarrow \mathscr{B})$ is a tautology. Hence if $\mathscr{A}_1$ and $\mathscr{B}_1$ are logically equivalent, then show that $\mathscr{A}$ and $\mathscr{B}$ are logically equivalent. 10

(b) *(i)* Find the statement form in the connectives ~, ∧ and ∨ that generates the function *f*, where

x_1	x_2	x_3	$f(x_1, x_2, x_3)$
T	T	T	T
T	F	T	T
T	T	F	T
T	F	F	F
F	T	T	F
F	F	T	F
F	T	F	F
F	F	F	T

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(ii) Show that the statement form $(p \rightarrow q) \wedge (q \rightarrow r) \rightarrow (p \rightarrow r)$ is a tautology. 5

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