

2017

MATHEMATICS

(Major)

Paper : 6.2

(Numerical Analysis)

Full Marks : 60

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

1. Answer the following questions : 1×7=7
- (a) If x is the true value of a quantity and x' is its approximate value, then what is the relative error?
 - (b) Define 'absolute error'.
 - (c) Define 'truncation error'.
 - (d) Evaluate $\Delta^2 a e^{bx}$.
 - (e) Write down the Weddle's rule.

(f) Write the relationship between the operator Δ and the differential operator D .

(g) Evaluate

$$\Delta^{10}(1-ax)(1-bx^2)(1-cx^3)(1-dx^4)$$

the interval of differencing being unity.

2. Answer the following questions : 2×4=8

(a) An approximate value of π is given by 3.1428571 and its true value is 3.1415926. Find the relative error.

(b) Show that

$$\Delta \log f(x) = \log \left\{ 1 + \frac{\Delta f(x)}{f(x)} \right\}$$

(c) What is the degree of the approximating polynomial corresponding to Simpson's $\frac{3}{8}$ th rule and Weddle's rule.

(d) Explain briefly the idea of numerical integration.

3. Answer the following questions :

(a) Round off the number 75462 to four significant digits and then calculate the absolute error and percentage error. 1+4=5

(b) Use the method of separation of symbols to prove the following identity :

$$u_1x + u_2x^2 + u_3x^3 + \dots = \frac{x}{1-x}u_1 + \frac{x^2}{(1-x)^2}\Delta u_1 + \frac{x^3}{(1-x)^3}\Delta^2 u_1 + \dots$$

where $u_{x+h} = E^h u_x$.

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(c) Derive Lagrange's interpolation formula and mention two advantages of it. 3+2=5

Or

Use Newton's forward interpolation formula

$$y = y_0 + u\Delta y_0 + \frac{u(u-1)}{2}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3}\Delta^3 y_0 + \dots$$

where $u = \frac{x-x_0}{h}$ to establish the formula

$$\left(\frac{d^2y}{dx^2}\right)_{x_0} = \frac{1}{h^2} \left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12} \Delta^4 y_0 - \frac{5}{6} \Delta^5 y_0 + \frac{137}{180} \Delta^6 y_0 + \dots \right]$$

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4. Answer either (a) or (b) :

(a) (i) What do you mean by 'divided difference'? Show that the divided differences are independent of the order of the arguments. $1+4=5$

(ii) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ using Simpson's $\frac{3}{8}$ th rule. 5

(b) (i) What do you mean by 'numerical differentiation'? Explain briefly with the help of a suitable example. $2+3=5$

(ii) Using Gauss's forward formula, find y for $x = 3.75$, given that

x :	2.5	3.0	3.5	4.0	4.5	5.0
y :	24.145	22.043	20.225	18.644	17.262	16.047

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5. Answer either (a) or (b) :

(a) (i) Find $f'(6)$ from the following table : 5

x :	0	1	3	4	5	7	9
$f(x)$:	150	108	0	-54	-100	-144	-84

(ii) The velocity v (km/min) of a car which starts from rest, is given at fixed intervals of time t (min) as follows :

t	:	2	4	6	8	10	12	14	16	18	20
v	:	10	18	25	29	32	20	11	5	2	0

Estimate approximately the distance covered in 20 minutes.

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(b) (i) Compute the integral $\int_5^{12} \frac{dx}{x}$ by applying Gauss's quadrature formula. Also find the error. $4+1=5$

(ii) Derive Newton-Cotes quadrature formula. How can the trapezoidal rule be deduced from it? $3+2=5$

6. Answer either (a) or (b) :

(a) (i) Derive the Newton-Raphson formula and show that the rate of convergence is quadratic. $3+2=5$

(ii) Find an approximate root of the equation $x^3 - 4x - 9 = 0$ using the bisection method four times. 5

- (b) (i) Explain the bisection method with suitable diagram. 5
- (ii) Find the root of the equation $xe^x = \cos x$ using secant method, correct to four decimal places (take the initial approximations as 0 and 1). 5
