2017

MATHEMATICS

(Major)

Paper : 6.1

(Hydrostatics)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×7=7

(a) What is the property of fluid pressure on which car brakes are designed?

(b) What do you mean by resultant vertical thrust on any surface of a homogeneous liquid at rest under the action of gravity?

(c) What are the necessary and sufficient conditions for a body to float freely wholly or partially immersed in the fluid under the action of gravity only?

(d) What do you mean by stable and unstable equilibrium of floating body?

(e) Define specific heat of a body.

(f) What is perfect gas?

(g) How is elasticity of a fluid measured?

2. Answer the following questions : 2×4=8

(a) Find the differential equations of the lines of force at any point (x, y, z).

(b) Prove that the position of the centre of pressure of a plane area is independent of the inclination of the area to the vertical.

(c) State the relations between pressure, density and temperature for a given mass of gas under different conditions.

(d) Define meta-centre and meta-centric height of a floating body.

3. Answer any *three* of the following : 5×3=15

(a) A liquid of given volume V is at rest under the forces

$$X = -\frac{\mu x}{a^2},\ Y = -\frac{\mu y}{b^2},\ Z = -\frac{\mu z}{c^2}$$

Find the pressure at any point of the liquid and the surfaces of equal pressure.

(b) An open vessel containing liquid is made to revolve about a vertical axis with uniform angular velocity. Find the form of the vessel and its dimensions that it may be just emptied.

(c) An ellipse is completely immersed with its minor axis horizontal and at a depth h. Find the position of the centre of pressure.

(d) Show that the equilibrium is stable or unstable according as the metacentre is above or below the centre of gravity of the body.

(e) For a perfect gas, establish the relation $C_p - C_v = R$, where the symbols have their usual notations.

4. Answer either *(a) or (b)* :

(a) *(i)* A closed tube in the form of an ellipse with its major axis vertical is filled with three different liquids of densities ρ_1, ρ_2, ρ_3 respectively. If the distances of the surfaces of separation from either focus be r_1, r_2, r_3 respectively, prove that

$$r_1(\rho_2 - \rho_3) + r_2(\rho_3 - \rho_1) + r_3(\rho_1 - \rho_2) = 0$$ 5

(ii) If a mass of a fluid, elastic or non-elastic, homogeneous or heterogeneous is at rest under the action of given forces, obtain the equation which determines the pressure at any point of the fluid. 5

(b) *(i)* If

$$X = y(y+z),\ Y = z(z+x),\ Z = y(y-x)$$

surfaces of equal pressure are the hyperbolic paraboloids $y(x+z) = c(y+z)$ and the curves of equal pressure and density are given by $y(x+z) = \text{constant}$, $y+z = \text{constant}$. 5

(Continued)

(ii) A given volume of liquid is at rest on a fixed plane under the action of a force, to a fixed point in the plane varying as the distance. Find the pressure at any point of the liquid and the whole pressure on the fixed plane. 5

5. Answer either (a) or (b) :

(a) (i) Show that the centre of pressure of a circular area immersed in the liquid whose centre is at a depth h below the surface, when the density of the liquid varies as the depth, is at a depth

$$\frac{2a^2h}{a^2+4h^2}$$

below the centre of the circle. 5

(ii) A hemispherical bowl is filled with water, and two vertical planes are drawn through its central radius, cutting off a semi-line of the surface; if 2α be the angle between the planes, prove that the angle which the resultant pressure on the surface makes with the vertical is

$$\tan^{-1}\left(\frac{\sin\alpha}{\alpha}\right)$$ 5

(b) (i) Determine the depth of the centre of pressure of a triangle in terms of the depths of its three vertices when one angular point A is in the free surface and the base is horizontal. 5

(ii) Find the resultant pressure on any surface of a fluid at rest under the action of any given force. 5

Or

A closed cylinder, very nearly filled with liquid, rotates uniformly about a generating line, which is vertical; find the resultant pressure on its curved surface.

6. Answer either *(a)* or *(b)* :

(a) (i) A solid cone is floating with its axis vertical and vertex downwards. Discuss its stability of equilibrium. 5

(ii) A cylindrical well of depth h and section A is maintained at constant temperature; if ρ_0 and ρ_1 are the densities of the air at the top and bottom, show that the total amount of air contained is

$$\frac{Ah(\rho_1 - \rho_0)}{\log \rho_1 - \log \rho_0}$$ 5

(b) (i) Find the position of equilibrium of a square lamina floating with its plane vertical in a liquid of double its own density. 5

(ii) Find the work done in compressing a gas in both cases when the change is isothermal and adiabatic. 5

Or

If the law connecting the pressure and density of the air were $p = k\rho^n$, prove that neglecting variations of gravity and temperature, the height of the atmosphere would be

$$\frac{n}{n-1}$$

times the homogeneous atmosphere.

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