2017

MATHEMATICS

(Major)

Paper : 4.2

(Mechanics)

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×10=10

(a) Define a couple.

(b) What is the graphical representation of the moment of a force?

(c) Can a force and a couple in the same plane be equivalent to a single force?

(d) Define limiting friction and limiting equilibrium.

(e) What is the centre of gravity of a triangular lamina?

(f) What is Poinsot's central axis?

(g) Define simple harmonic motion.

(h) What are the characteristics of a central orbit?

(i) Define null lines and null planes.

(j) State the principle of virtual work.

2. Answer the following questions : 2×5=10

(a) Derive a relation between linear velocity and angular velocity.

(b) The law of motion in a straight line is $s = \frac{1}{2}vt$. Prove that the acceleration is constant.

(c) What are the general conditions of equilibrium of any system of coplanar forces?

(d) State the laws of statical friction.

(e) Find the CG of an arc of a plane curve $y = f(x)$.

3. Answer any *four* of the following questions : 5×4=20

(a) Prove that any system of coplanar forces acting on a rigid body can ultimately be reduced to a single force acting at any arbitrarily chosen point in the plane, together with a couple.

(b) Find the centre of gravity of the area of the cardioid $r = a(1 + \cos\theta)$.

(c) State the forces which may be omitted in forming the equation of virtual work.

(d) Derive the expressions for the tangential and normal components of the acceleration of a particle describing a plane.

(e) A particle is moving in a straight line with an acceleration n^2(distance) towards a fixed point in the line, in a medium which offers a resistance proportional to velocity and is simultaneously acted on by a periodic disturbing force $F \cos pt$ per unit mass.

(f) A particle moves is an ellipse under a force which is always directed to a focus. Find the law of force, the velocity and the periodic time.

4. (a) A regular hexagon is composed of six equal heavy rods freely joined together, and two opposite angles are connected by a string, which is horizontal. One rod being in contact with a horizontal plane; at the middle point of opposite rod is placed a weight W_1; if W be the weight of each rod, show that the tension of the string is $\frac{3W + W_1}{\sqrt{3}}$. 5

Or

Whatever may be the base or the axes chosen to reduce the system, show that the quantities $(LX + MY + NZ)$ and $(X^2 + Y^2 + Z^2)$ are invariants for any given system of forces. The symbols have their usual meanings.

(b) State the Kepler's laws of planetary motion and briefly mention their dynamical significance. 5

5. *(a)* If an arc of a plane curve rotates through any angle about an axis in its plane which does not cross it, then the area of the system generated by revolving arc is equal to the length of the arc multiplied by the length of the path described by the centre of gravity. 5

(b) A perfectly rough body rests in equilibrium on a fixed body which is convex upwards. Find whether the equilibrium is stable or unstable. 5

6. *(a)* Prove that a given system of forces can have only one central axis. 5

Or

Two forces act, one along the line $y = 0$, $z = 0$ and the other along the line $x = 0$, $z = c$. As the forces vary, show that the surface generated by the axis of their equivalent wrench is $(x^2 + y^2)z = cy^2$.

(b) A particle is tied to one end of an elastic string, the other end of which is tied to a fixed point on a smooth table. The particle is pulled out to a certain distance from the fixed point and let go. Discuss the motion. 5

7. *(a)* A particle is describing a circle of radius a in such a way that its tangential acceleration is k times the normal acceleration, where k is a constant. If the speed of the particle at any point be u, prove that it will return to the same point after a time

$$\frac{a}{ku}(1-e^{-2\pi k})$$ 5

Or

A spherical rain drop, falling freely receives in each instant an increase of volume equal to λ time its surface at that instant. Show that the velocity at the end of time t is given by

$$\frac{g}{4\lambda}\left\{\frac{(a+\lambda t)-a^4}{(a+\lambda t)^3}\right\}$$

and the distance fallen through in that time is

$$g\frac{t^2}{8}\left\{\frac{2a+\lambda t}{a+\lambda t}\right\}^2$$

(b) If two forces act a point, the algebraic sum of their moments about any line in space is equal to the moment of their resultant about the same line. 5

Or

If a planet was suddenly stopped in its orbit supposed circular, show that it would fall into the sun in a time which is $\sqrt{\left(\frac{2}{8}\right)}$ times the period of the planet's revolution.

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