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MATHEMATICS

(Major)

Paper : 2.2

(Differential Equation)

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following : 1×10=10

- (a) Determine the order and degree of the differential equation

$$K \frac{d^2 y}{dx^2} = \left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{3/2}$$

- (b) Define Bernoulli's differential equation.

- (c) If the differential equation $Mdx + Ndy = 0$ is homogeneous and $Mx + Ny \neq 0$, write the integrating factor.

- (d) What do you mean by self-orthogonal family of curves?

(2)

- (e) What is the complementary function of the following differential equation?

$$(D^3 + 3D^2 + 3D + 1)y = e^{-x}$$

- (f) Write down the general solution of the differential equation

$$y = px + p - p^2$$

- (g) Write down the condition of exactness of a total differential equation

$$Pdx + Qdy + Rdz = 0$$

- (h) Find the particular integral of the differential equation

$$(D^2 + a^2)y = \sin ax$$

- (i) Write the standard form of the linear partial differential equation of order one.

- (j) Find an integral belonging to complementary function of the differential equation

$$y_2 - \cot xy_1 - (1 - \cot x)y = e^x \sin x$$

2. Answer the following questions : 2×5=10

- (a) Form the differential equation of which $xy = ae^x + be^{-x}$ (a ; b parameters) is a solution.

(b) Solve :

$$(x+y)^2 \frac{dy}{dx} = a^2$$

(c) Solve :

$$\frac{dx}{y^2} = \frac{dy}{x^2} = \frac{dz}{x^2 y^2 z^2}$$

(d) If $\frac{dy}{dx} + 2y \tan x = \sin x$ and if $y=0$

when $x = \frac{\pi}{2}$ express y in terms of x .

(e) Construct the partial differential equation by eliminating a and b from

$$z = (x^2 + a)(y^2 + b)$$

3. Answer any four questions :

5×4=20

(a) Prove that a necessary and sufficient condition that the differential equation $Mdx + Ndy = 0$ be exact is that

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

(b) Find the orthogonal trajectories of the family of curves $x^2 + y^2 = 2ax$, a being parameter.

(c) Solve :

$$\frac{dy}{dx} = \frac{3y - 7x + 7}{3x - 7y - 3}$$

- (d) Apply variation of parameter to solve the differential equation

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - y = x^2 e^x$$

- (e) Solve :

$$x^3 \frac{d^3 y}{dx^3} + 6x^2 \frac{d^2 y}{dx^2} + 8x \frac{dy}{dx} + 2y = x^2 + 3x - 4$$

- (f) Obtain the general and singular solution of the differential equation

$$y = px + \sqrt{b^2 + a^2 p^2}$$

4. Answer either (a) and (b) or (c) and (d) : 5+5=10

- (a) Solve :

$$\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + 4y = e^x \cos x$$

- (b) Solve :

$$t dx = (t - 2x) dt$$

$$t dy = (tx + ty + 2x - t) dt$$

- (c) Solve :

$$\frac{dy}{dx} = x^3 y^3 - xy$$

- (d) Reduce the equation $y^2(y - px) = x^4 p^2$, where $p = \frac{dy}{dx}$ to Clairaut's form by the substitution $x = \frac{1}{X}$, $y = \frac{1}{Y}$ and hence solve the equation.

5. Answer either (a) and (b) or (c) and (d) : 5+5=10

- (a) Find the necessary condition for integrability of the total differential equation $Pdx + Qdy + Rdz = 0$.

- (b) Reduce the differential equation

$$\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = R(x)$$

to the form $\frac{d^2v}{dx^2} + Q_1v = R_1$, where Q_1

and R_1 are functions of x to solve the differential equation.

- (c) Solve :

$$x \frac{d^2y}{dx^2} - \frac{dy}{dx} - 4x^3y = 8x^3 \sin x^2$$

by changing the independent variable x to z .

- (d) Solve :

$$(1 - x^2) \frac{d^2y}{dx^2} + x \frac{dy}{dx} - y = x(1 - x^2)^{3/2}$$

6. Answer either (a) and (b) or (c) and (d) : 5+5=10

(a) Solve :

$$\frac{dx}{x^2 - y^2 - z^2} = \frac{dy}{2xy} = \frac{dz}{2xz}$$

(b) Solve :

$$3x^2 dx + 3y^2 dy - (x^3 + y^3 + e^{2z}) dz = 0$$

(c) Show that the differential equation

$$x dx + y dy = \frac{a^2(x dy - y dx)}{x^2 + y^2}$$

is exact and hence solve it.

(d) Find $f(z)$ such that

$$\left(\frac{y^2 + z^2 - x^2}{2x} \right) dx - y dy + f(z) dz = 0$$

is integrable and hence solve it.

7. Answer either (a) and (b) or (c) and (d) : 5+5=10

(a) Solve by Lagrange's method

$$z(x+y)p + z(x-y)q = x^2 + y^2$$

(b) Solve by Charpit's method

$$pxy + pq + qy - yz = 0$$

(7)

- (c) Find the complete integral of $9(p^2z + q^2) = 4$. Also find the singular solution if it exists.
- (d) Derive the partial differential equation by the elimination of arbitrary function from the equation $\phi(u, v) = 0$, where u and v are functions of x , y and z .

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