

**3 (Sem-2) MAT M 1****2 0 1 7****MATHEMATICS****( Major )****Paper : 2-1****( Coordinate Geometry )****Full Marks : 80****Time : 3 hours**

*The figures in the margin indicate full marks  
for the questions*

1. Answer the following questions : 1×10=10

(a) Transform to axes inclined at  $45^\circ$  to the original axes the equation  $x^2 - y^2 = a^2$ .

(b) Write down the condition for pair of lines represented by the equation

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

(c) Write down the parametric equations of the parabola.

(d) Write down the direction cosines of x-axis.

(e) About which axis the parabola  $y^2 = 4ax$  is symmetric?

(f) Find the eccentricity of the ellipse

$$x^2 + 3y^2 = a^2$$

(g) Write the equation of the diameter of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

parallel to the line  $y = mx + c$ .

(h) Find the centre and radius of the sphere

$$x^2 + y^2 + z^2 - 2x + 4y + 2z + 3 = 0$$

(i) Define conjugate planes.

(j) Define enveloping cylinder.

2. Answer the following :

2×5=10

(a) If the axes be turned through an angle  $\tan^{-1} 2$ , what does the equation  $4xy - 3x^2 = a^2$  become?

(b) Find the value of  $k$  so that  $kxy - 8x + 9y = 12$  may represent pair of straight lines.

- (c) Find the equation of the sphere through circles

$$x^2 + y^2 + z^2 = 25, \quad x + 2y - z + 2 = 0$$

and the point (1, 1, 1).

- (d) If  $(at_1^2, 2at_1)$  and  $(at_2^2, 2at_2)$  are the extremities of any focal chord of the parabola  $y^2 = 4ax$ , prove that  $t_1 t_2 = -1$ .

- (e) The axis of a right circular cylinder is

$$\frac{x-1}{2} = \frac{y-2}{-1} = \frac{z-3}{2}$$

and its radius is 5. Find the equations.

3. Answer any *two* parts :

5×2=10

- (a) By transforming to parallel axes through a properly chosen point  $(h, k)$ , prove that the equation

$$12x^2 - 10xy + 2y^2 + 11x - 5y + 2 = 0$$

can be reduced to one containing only terms of the second degree.

- (b) Reduce the equation

$$7x^2 - 2xy + 7y^2 - 16x + 16y - 8 = 0$$

to the standard form.

(c) Show that the equation

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

represents a pair of parallel straight lines, if

$$\frac{a}{h} = \frac{h}{b} = \frac{g}{f}$$

(d) Prove that the sum of the squares of the reciprocals of two perpendicular diameters of an ellipse is constant.

4. Answer any two parts :

5×2=10

(a) Prove that the straight line  $y = mx + c$  touches the parabola  $y^2 = 4a(x + a)$ , if

$$c = ma + \frac{a}{m}$$

(b) Find the length of the semi-axes of the conic  $ax^2 + 2hxy + ay^2 = d$ .

(c) Prove that the line  $lx + my = n$  is a normal to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\text{if } \frac{a^2}{l^2} + \frac{b^2}{m^2} = \frac{(a^2 - b^2)^2}{n^2}$$

- (d) Find the equation of the tangent to the hyperbola  $4x^2 - 9y^2 = 1$  which is parallel to the line  $4y = 5x + 7$ .

5. Answer any four parts : 5×4=20

- (a) Find the condition that the homogeneous equation of second degree  $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$  may represent a pair of planes.

- (b) Obtain the shortest distance between the lines

$$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$$

and

$$\frac{x-\alpha'}{l'} = \frac{y-\beta'}{m'} = \frac{z-\gamma'}{n'}$$

- (c) Prove that the equation of the plane containing the line

$$\frac{y}{b} + \frac{z}{c} = 1, \quad x = 0$$

and parallel to the line  $\frac{x}{a} - \frac{z}{c} = 1, y = 0$  is

$$\frac{x}{a} - \frac{y}{b} - \frac{z}{c} + 1 = 0$$

If  $2d$  is the shortest distance between the given lines, prove that

$$\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{d^2}$$

- (d) Find the condition when the plane  $lx + my + nz = p$  becomes a tangent plane to the sphere

$$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$$

- (e) Find the equation of the cone whose vertex is at the origin.

- (f) Prove that the equation of the polar of the origin with respect to the conic

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

$$\text{is } gx + fy + c = 0.$$

6. Answer any four parts : 5×4=20

- (a) Find the equation of the cylinder whose axis and guiding curve are given.

- (b) Find the condition when the plane  $lx + my + nz = p$  becomes a tangent plane to the conicoid  $ax^2 + by^2 + cz^2 = 1$ .

- (c) Find the equation of the enveloping cone of a conicoid whose vertex is given.

- (d) If the section of the enveloping cone of the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

whose vertex is  $P$ , by the plane  $z = 0$  is a rectangular hyperbola, prove that the locus of  $P$  is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

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- (e) Prove that the locus of the poles of tangent plane of the conicoid  $ax^2 + by^2 + cz^2 = 1$  with respect to the conicoid  $\alpha x^2 + \beta y^2 + \gamma z^2 = 1$  is the conicoid

$$\frac{\alpha^2 x^2}{a} + \frac{\beta^2 y^2}{b} + \frac{\gamma^2 z^2}{c} = 1$$

- (f) Find the equation of the right circular cylinder whose guiding curve is

$$x^2 + y^2 + z^2 = 9$$

$$x - y + z = 3$$

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