

3 (Sem-2) PHY M 1

2 0 1 6

PHYSICS

(Major)

Paper : 2.1

Full Marks : 60

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

GROUP—A

(Mathematical Methods—II)

(Marks : 35)

1. Answer the following questions : 1×4=4

(a) Find the value of \vec{r} satisfying the equation $\left(\frac{d^2 \vec{r}}{dt^2} \right) = a$.

(b) What are the coordinate surfaces in cylindrical coordinates?

(c) Find the value of $\Gamma(0)$.

(d) Evaluate : $\int_2^6 (3x^2 - 2x - 1) \delta(x - 3) dx$

2. Answer the following questions : 2×3=6

(a) Find the total work done in moving a particle in a force field given by $\vec{F} = 3xy\hat{i} - 5z\hat{j} + 10x\hat{k}$ from $t=1$ to $t=2$.

(b) Define Gamma function and Dirac delta function.

(c) Show that $\iint_S \vec{A} \cdot \hat{n} dS$, over any closed

surface S is equal to $\iint_R \vec{A} \cdot \hat{n} \frac{dx dy}{|\hat{n} \cdot \hat{k}|}$, where

R is the projection of S on xy -plane.

3. Answer any two of the following questions :

3×2=6

(a) If $\vec{F} = (2x^2 - 3z)\hat{i} - 2xy\hat{j} - 4x\hat{k}$, then evaluate $\iiint_V \vec{\nabla} \cdot \vec{F} dV$, where V is the region bounded by the planes $x=0$, $y=0$, $z=0$ and $2x+2y+z=4$.

(b) Show that in orthogonal coordinates

$$\vec{\nabla} \times (A_1 \hat{e}_1) = \frac{\hat{e}_2}{h_3 h_1} \frac{\partial}{\partial u_3} (A_1 h_1) - \frac{\hat{e}_3}{h_1 h_2} \frac{\partial}{\partial u_2} (A_1 h_1)$$

where symbols have their usual meanings.

(c) Show that $\Gamma(n) = (n-1)\Gamma(n-1)$ for all values of n and $\Gamma(n) = (n-1)!$, when n is a positive integer.

4. Answer any one of the following : 4

(a) Verify divergence theorem for the vectors $\vec{V} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$, taken over the cube $0 < x, y, z < 1$.

(b) Prove that a spherical coordinate system is orthogonal.

5. Answer any three questions from the following : 5×3=15

(a) Show that—

(i) $\delta(-x) = \delta(x)$

(ii) $x\delta'(x) = -\delta(x)$ 2+3=5

(b) Show that $\iint_S (\vec{\nabla} \times \vec{A}) \cdot \hat{n} dS = \oint_C \vec{A} \cdot d\vec{r}$

(c) Express $\nabla^2\psi$ in orthogonal curvilinear coordinates.

(d) If S be a closed surface and \vec{r} denotes the position vector of any point (x, y, z) measured from an origin O , prove that

$$\iint_S \frac{\hat{n} \cdot \vec{r}}{r^3} dS \text{ is equal to } 4\pi \text{ if } O \text{ lies inside } S.$$

(e) Find the circulation of F round the curve C , where

$$\vec{F} = e^x \sin y \hat{i} + e^x \cos y \hat{j}$$

and C is the rectangle whose vertices are $(0, 0)$, $(1, 0)$, $(1, \frac{\pi}{2})$ and $(0, \frac{\pi}{2})$.

(4)

GROUP—B

(Properties of Matter)

(Marks : 25)

6. Answer the following questions : 1×5=5

- (a) Why is a hollow shaft of the same material, mass and length very much stronger than that of a solid shaft?
- (b) What is the velocity profile of a advancing liquid through a horizontal capillary tube?
- (c) When is the surface energy of a liquid surface equal to the surface tension?
- (d) Write the expression for excess pressure of cylindrical bubble in a liquid.
- (e) Write the expression for time period of a torsional pendulum.

7. Answer either (a) and (b) or (c) and (d) : 10

Either

- (a) Find an expression for energy per unit volume of a stretching wire. Show that a shear is equivalent to a compression and an extension. 3+3=6

- (b) A bronze bar, 1.7 m long and 50 mm in diameter is subjected to a tensile stress of 70 meganewton/m². Calculate the extension produced in the bar and the work done during the process. The value of Young's modulus for the material of the bar may be taken to be $85 \times 10^9 \text{ N/m}^2$. 4

Or

- (c) Calculate the depression at the free end of a thin light beam, clamped horizontally at one end and loaded at the other. 5
- (d) For the same cross-sectional area, show that a beam of square section is stiffer than that of a circular section of same material. Show also for a given load the depressions are in the ratio $3 : \pi$. 5

8. Answer any *two* questions of the following :

$5 \times 2 = 10$

- (a) Show that shearing stress at a point in a twisted cylinder or a wire depends on the distance of the point from the axis and not its vertical distance from either end of it. 5

(b) The rate of flow of liquid through a capillary is given $Q = \frac{\pi Pr^4}{8\eta l}$ with usual notations. Deduce this relation stating clearly the conditions under which it holds. Why does the formula fail in the case of tube of wide bore? 4+1=5

(c) (i) For a homogeneous isotropic substance, show that $\frac{Y}{\eta} = 2(\sigma + 1)$, where symbols have their usual meanings.

(ii) A gold wire 0.32 mm in diameter elongated by 1 mm, when stretched by a force of 330 gm-wt and twists through 1 radian, when equal and opposite torque of 145 dyne-cm are applied at its end. Find the value of Poisson's ratio for gold. 3+2=5

(d) (i) Show that excess pressure acting on a curved surface of a curved membrane is given by

$$P = 2T \left(\frac{1}{r_1} + \frac{1}{r_2} \right)$$

where r_1 and r_2 are the radii of curvature and T the surface tension of the membrane.

(7)

- (ii) Two equal spherical soap bubbles coalesce to form one spherical soap bubble without any leakage of air. If V is the consequence change in volume of the contained air and S , the change in the total surface area, show that $3PV + 4ST = 0$, where T is the surface tension of the soap bubble and P , the atmospheric pressure. 3+2=5
