2016

PHYSICS

(Major)

Paper : 5.1

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Mathematical Methods)

(*Marks* : 30)

1. Answer the following questions : 1×4=4

   (a) Find the real part of $\frac{1+z}{1-z}$.

   (b) What is the argument of $-3i$?

   (c) Define pole and residue.

   (d) Find the principal value of i^i.

2. *(a)* Find the complex conjugate of the functions

   $$(x+iy)\cdot(a+ib) \text{ and } \frac{x-iy}{a+ib}$$

   where x, y, a and b are real. 4

   (b) Obtain the modulus of the complex number $\frac{1-i}{1+i}$. 2

(Turn Over)

3. (a) State De Moivre's theorem. 2

(b) Using De Moivre's formula, evaluate

$$(\cos 20° + i \sin 20°)^9$$ 2

4. (a) Define equivalent contour. 2

(b) Verify if the function $f(z) = z$ is analytic. 4

Or

(i) Determine if the function e^{iz} is analytic. 2

(ii) Prove that

$$\left(\frac{z_1}{z_2}\right) = \arg z_1 - \arg z_2$$ 2

5. (a) Check the analyticity and hence find the derivative of the function $f(z) = \sin z$. 5

Or

Find Taylor series expansion about the origin for $f(z) = \ln(1 + z)$. 5

(b) Find Laurent expansion for the function $f(z) = \frac{\sin z}{z^4}$ about $z_0 = 0$ and hence classify the singularity and calculate the residue. 5

Or

State and prove Cauchy's integral theorem. 5

GROUP—B

(Classical Mechanics)

(*Marks* : 30)

6. Answer the following questions : $1\times3=3$

(a) State the principle of virtual work.

(b) Define central force and write down a general expression for it.

(c) Define Hamiltonian of a system.

7. Answer any *three* of the following questions : $2\times3=6$

(a) Explain with example the meaning of holonomic constraint.

(b) Show that angular momentum is a constant of central force motion.

(c) In a two-body system, the masses are in the ratio 4 : 1. The mass of the lighter body is 10^{-28} g. Estimate the reduced mass of the system.

(d) What are generalized coordinates and generalized velocities?

8. Answer any *four* of the following questions : $4\times4=16$

(a) Show that a two-body central force problem can be reduced to one-body problem.

(b) Find the equation of motion of a system with the given Lagrangian

$$L = \frac{1}{2} e^{\alpha t} (\dot{x}^2 - \omega^2 x^2)$$

where α and ω are constants.

(c) Obtain the general differential equation of a cental orbit.

(d) Show that if the Lagrangian function does not contain the time explicitly, the total energy of the conservative system is conserved.

(e) Construct the Lagrangian and hence equation of motion of a simple pendulum placed in a uniform gravitational field.

(f) Set up Lagrangian equation for an Atwood machine and find an expression for its acceleration.

9. Establish Hamilton's canonical equations. 5

Or

Obtain Lagrange's equation of motion for a conservative system using D'Alembert's principle. 5

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