2016

PHYSICS

(Major)

Paper : 4.1

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Mathematical Methods—IV)

1. Answer any *four* of the following questions : 1×4=4

(a) Find the value of n if

$$\int_{-1}^{+1} P_n(x)\,dx = 2$$

(b) Write the general solution of the equation

$$y'' + 2y' + 5y = 0$$

(c) Define Gaussian distribution.

(d) What is singular point in a second-order linear differential equation?

(e) If $H_n(x)$ is the Hermite polynomial, then what is the value of $H_0(x)$?

(f) What do you mean by standard deviation?

2. Answer any *three* of the following questions : 2×3=6

(a) Check whether Frobenius method can be applied or not to the following equation :

$$\frac{d^2y}{dx^2} - \frac{5y}{x^3} = 0$$

(b) Find the value of $P_{2n+1}(0)$.

(c) When one card is drawn from each of two decks, find the probability that at least one of them is an ace.

(d) Find the degree and order of the following equation :

$$\frac{d^2y}{dx^2}+\frac{2}{x}\frac{dy}{dx}+\frac{9^2}{x^4}=0$$

(e) Prove that $H_1(x)=2x$.

3. Answer any *two* of the following questions : 5×2=10

(a) Use Frobenius method to find the series solution of the equation

$$\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(x^2+2)y=0$$

(b) Show that

$$\int_{-1}^{+1}P_m(x)\,P_n(x)\,dx=0$$

where $m\neq n$.

(c) Write the generating function for Hermite polynomial $H_n(x)$ and hence show that

$$H_n(x)=(-1)^n e^{x^2}\frac{d^n}{dx^n}e^{-x^2}$$

(d) A manufacturer produces air mail envelope whose weight is normally distributed with $\mu = 1{\cdot}95$ g and standard deviation $\sigma = 0{\cdot}05$ g. The envelopes are sold in lots of 1000. How many envelopes in a lot will be heavier than 2g? Use the fact

$$\frac{1}{\sqrt{2n}}\int_0^1 e^{-x^2/a}\,dx = 0{\cdot}3413$$

(e) Show that

$$\int_{-\infty}^{+\infty} e^{-x^2} H_m(x)\, H_n(x)\,dx = 0$$

if $m \neq n$.

4. Answer any *two* of the following questions : 10×2=20

(a) (i) Show that

$$(1-2xh+h^2)^{-1/2} = \sum_{n=0}^{\infty} P_n(x)\,h^n$$

where $P_n(x)$ is the Legendre polynomial. 5

(ii) Evaluate explicitly the Legendre Polynomials $P_2(x)$ and $P_3(x)$. 2½+2½=5

(b) Prove the following recurrence relations :

(i) $2xH_n(x) = 2nH_{n-1}(x) + H_{n+1}(x)$ 3

(ii) $H_n'(x) = 2xH_n(x) - H_{n+1}(x)$ 3

(iii) $2nH_{n-1}(x) = H_n'(x)$ 4

(c) (i) A dice is thrown 8 times. Find the probability that '5' will show exactly twice, at least seven times and at least once. 3+3+2=8

(ii) Define total probability. 2

(d) Obtain the power series solution of the Legendre equation

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0 \qquad 10$$

GROUP—B

(Introduction to Computer and Computer Programming)

5. Answer any *two* of the following : 1×2=2

(a) How will you test the efficiency of an algorithm?

(b) Name the two units of the central processing unit.

(c) What is a variable?

6. Answer any *two* of the following : 2×2=4

(a) What are the fundamental data types in C^{++}/C/FORTRAN?

(b) What is the syntax to find the number of characters in a string in C^{++}/C/FORTRAN?

(c) What are the advantages of breaking a program into subroutine?

7. Answer any *one* of the following : 4

(a) Write a note on high-level language. What are its advantages?

(b) What are the basic major computer operations or functions?

(Continued)

8. Answer any *one* of the following :

(a) Write the algorithm and draw the flow chart to find a prime number from a set of numbers. 5+5=10

(b) Define transfer statement. What are the different types of transfer statements? Write the syntax of all the transfer statements in C^{++}/C/FORTRAN. 1+3+6=10

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